**AFM 472**

**Midterm Examination**

**Monday Oct. 24, 2011**

**A. Huang**

**Name: Answer Key**

**Student Number:** ______________________________

**Section (circle one): 10:00am 1:00pm 2:30pm**

**Instructions:**

1. **Answer all questions in the space provided. If space is not enough, use the back of the page.**
2. **Show all of your calculations.**
3. **The examination has 15 pages (not including this cover page). Verify that your copy is complete.**
4. **Materials allowed: calculator.**
5. **Unless specifically instructed otherwise, provide final answers relating to percentage rates to four decimal places (e.g. 6.27% or .0627) and provide final answers involving dollar amounts to two decimal places (e.g. $98.27).**
6. **To have your exam considered for re-grading, the exam must be written in ink.**
7. **Pages 16 and 17 of the exam are a formula sheet. Do not write any part of your answers on these pages. It will not be graded. If you find it easier to consult these pages by detaching it from your exam, please do so. You are not expected to hand in the formula sheet.**

**Mark Distribution**

| | |
|---|---|
| **1. ________/16** | **5. ________/13** |
| **2. ________/10** | **6. ________/13** |
| **3. ________/10** | **7. ________/14** |
| **4. ________/13** | **8. ________/11** |

**Total: ________/100**

Question 1—Multiple choice (2 marks each, 16 marks in total): Circle one answer that is the best.

(1) If the annual real rate of interest is 5% and the expected inflation rate is 4%, the nominal rate of interest would be approximately

a) 1%.
b) **9%.**
c) 20%.
d) 15%.
e) None of the above.

(2) In the Modern Portfolio Theory, which of the following statements about the global-minimum variance portfolio of all risky securities is valid? (Assume short sales are allowed.)

a) It must lie inside the mean-variance frontier.
b) **Its variance must be lower than those of all other securities or portfolios.** (source: Chapter 6, problem 3)
c) It may be optimal risky portfolio.
d) It must include all individual securities.
e) None of the above.

(3) Assume that stock BCE.TO has a Minimum Guaranteed Fill (MGF) of 1,000 shares at the Toronto Stock Exchange. The current quote for the stock is 800 $39.24 – $39.26 600, where 800 and 600 are number of shares. The next best bid is 700 shares at $39.23, and the next best ask is 600 shares at $39.27. We know that in TSX there is a market maker for the stock and that the MGF is in force. If a retail client orders to buy 1,000 shares with a market order, what will happen? The client buys:

a) 600 shares at $39.26 and 400 shares at $39.27.
b) 800 shares at $39.24 and 200 shares at $39.23.
c) **1,000 shares at $39.26.**
d) 800 shares at $39.24 and 200 shares at $39.26.
e) The client will trade directly with the market maker and accept whatever the price the market maker offers.

(4) Which of the following statements are TRUE?

I. The market portfolio consists of all the risky assets in the universe.
II. Securities that fall above the SML are underpriced.
III. Securities that fall below the SML are underpriced.
IV. Securities that fall on the SML have no intrinsic value of the investor.
V. The risk-free rate defines where the SML intersects the Y-axis.

a) I and III only
b) I, III and V only
c) **I, II, and V only**
d) I, II, IV, and V only.

(5) You are considering investing $1,000 in a T-bill that pays a rate of return of 0.05 and a risky portfolio, $P$, constructed with 2 risky securities, $X$ and $Y$. The weights of $X$ and $Y$ in $P$ are 0.60 and 0.40, respectively. $X$ has an expected rate of return of 0.14 and a variance of 0.01, and $Y$ has an expected rate of return of 0.10 and a variance of 0.0081. If you want to form a portfolio with an expected rate of return of 0.10, what percentages of your money must you invest in the T-bill, $X$, and $Y$, respectively if you keep $X$ and $Y$ in the same proportions to each other as in portfolio $P$?

a) 0.25; 0.45; 0.30
b) 0.19; 0.49; 0.32
c) **0.32; 0.41; 0.27**
d) 0.50; 0.30; 0.20
e) cannot be determined

(6) The change from a straight to a kinked capital allocation line is a result of the

a) Reward-to-variability ratio increasing
b) Investor's risk tolerance decreasing
c) **Borrowing rate exceeding the lending rate** (source: Chapter 5, problem 32)
d) Increase in the portfolio proportion of the risk-free asset
e) None of the above.

(7) You sold ABC stock short at $80 per share. Your losses could be minimized by placing a _________.

a) limit-sell order at $75.
b) limit-buy order at $79.
c) **stop-buy order at $85.**
d) stop-limit order to sell at $85.

(8) In an efficient market the correlation coefficient between stock returns for two non-overlapping time periods should be ________:

a) positive and large.
b) positive and small.
c) **zero.**
d) negative and small.
e) negative and large.

Question 2: 10 marks.

Consider the following limit-order book of a market-maker. The last trade in the stock took place at a price of \$30.00. Sub-questions (a)–(d) are independent of each other.

| Buy<br>Size (shares) | Price | Sell<br>Size (shares) |
|---|---|---|
| 200 | 26.87 | |
| 100 | 27.12 | |
| 200 | 28.50 | |
| 100 | 29.00 | |
| 100 | 29.75 | |
| | 30.25 | 200 |
| | 30.38 | 500 |
| | 30.50 | 300 |
| | 30.88 | 800 |

(a) (2 marks) If the next order is a market order to sell 200 shares, at what price will it be filled?

Answer: 100 shares at \$29.75 and 100 shares at \$29. (1 mark each)

(b) (3 marks) If the next order is a market-buy order for 100 shares, at what price will it be filled? What's the market after this order?

Answer: 100 shares at \$30.25. (1 pt). The market after that will be: 100 \$29.75 – \$30.25 100 (100 shares bid at \$29.75 t0 100 shares ask at \$30.25). (2 pt)

(c) (3 marks) Suppose 500 market sell order arrives, what will be the price jump? As the market maker of the stock and foreseeing such jumps, what can you do to reduce the volatility of the price?

Answer: The current price is \$30. The price jump will be: \$30 to \$29.75, and all the way to \$27.12 as market absorbs the 500 sell order. (1.5 pts). As the market maker, you can insert buy order in-between \$27.12 and \$29.75 to reduce volatility, say, place a buy order of 400 shares at \$29.50. (1.5 pts)

(d) (2 marks) You are considering placing a stop-buy order. Which of the following two stop prices, \$28.50 or \$32.50, makes more sense for your stop-buy order? Why (use one sentence)?

Answer: \$32.50. Stop-buy is used when increase of price triggers a need to buy stocks.

Question 3: 10 marks. This question is split to this and the next page. Sub-question (c) is independent of (a) and (b).

Imagine that there are only two pervasive macroeconomic factors. Stocks X, Y, and Z have the following sensitivities to these two factors:

| Stock | $\beta_1$ (Factor 1) | $\beta_2$ (Factor 2) |
|---|---|---|
| X | 1.25 | 0.75 |
| Y | −0.50 | 1.50 |
| Z | 3.00 | 2.00 |

Assume that the expected risk premium is 4% on Factor 1 and 8% on Factor 2. Treasury bills offer zero risk premium.

(a) (3 marks) According to the arbitrage pricing theory what is the risk premium on each of the three stocks?

Answer:

$$RP_X = 1.25(0.04) + 0.75(0.08) = 0.11$$

$$RP_Y = -0.50(0.04) + 1.50(0.08) = 0.10$$

$$RP_Z = 3.00(0.04) + 2.00(0.08) = 0.28$$

(b) (4 marks) Suppose you buy \$100 of X and \$50 of Y, and sell \$80 of Z. What is the sensitivity of your portfolio to each of the two factors? What is the expected risk premium of your portfolio?

Answer: Let $w_x$, $w_y$, and $(1 - w_x - w_y)$ be the weights on stocks X, Y, and Z.

Total investment $= 100 + 50 - 80 = \$70$

$$w_x = 100/70 = 1.4286; \quad w_y = 50/70 = 0.7143; \quad w_z = 1 - 1.4286 - 0.7143 = - - 1.1429$$

The sensitivities of this portfolio to these factors are

$$Factor1 : 1.4286(1.25) + 0.7143(-0.50) + (-1.1429)(3.00) = -2.0001 \qquad (1.5pt)$$

$$Factor2 : 1.4286(0.75) + 0.7143(1.50) + (-1.1429)(2.00) = -0.1429 \qquad (1.5pt)$$

Expected risk premium of the portfolio:

$$(-2.0001)(0.04) + (-0.1429)(0.08) = -0.0914 \qquad (1pt)$$

Question 3 cont'd:

(c) (3 marks) Short-answer: List and briefly illustrate three differences between CAPM and APT. For each difference, your illustration should not exceed two sentences.

Suggested answer:

| | CAPM | APT |
|---|---|---|
| Theory | Equilibrium model | No-arb. condition |
| What is the "market" portfolio | Mean-variance efficient<br>Unobservable market portfolio | Diversifiable portfolio |
| Factor(s) | Market portfolio | Unknown |

You can also talk about, for example, (1) assumption differences in those two theories, (for example, investors need not have homogeneous belief about expected returns in APT); (2) factors in APT (for example, market portfolio may not even be a factor in APT); and (3) investors are not constrained to be mean-variance optimizers in APT, etc.

Question 4: 13 marks. This question is split to this and the next page.

The current stock price of A is \$10, and stock price of B is \$5. You believe that their return distributions are as follows:

| Stock | Return in Recession (probability =.5) | Return in Boom (probability = .5) |
|---|---|---|
| A | $-5\%$ | $20\%$ |
| B | $2\%$ | $15\%$ |

(a) (6 marks) If these two stocks are to consist of the market portfolio, and you have \$100 to invest in these two stocks, how should you invest such that you are holding the optimal portfolio (i.e. when you try to achieve the maximal Sharpe ratio)? Assume that the riskfree rate is 4%. To minimize your calculation, the covariance between returns of A and B as implied by the above numbers is 0.008125.

Answer: Using the formula:

$$w_D = \frac{[E(r_D) - r_f]\sigma_E^2 - [E(r_E) - r_f]Cov(r_D,r_E)}{[E(r_D) - r_f]\sigma_E^2 + [E(r_E) - r_f]\sigma_D^2 - [E(r_D) + E(r_E) - 2r_f]Cov(r_D,r_E)}$$

and replace $D$, $E$ with A, B.

$$E(r_A) = 0.5(-5\%) + 0.5(20\%) = 7.5\%$$

$$\sigma_A = 12.5\%$$

(Note that the standard deviation is the distance between 20% to the mean of 7.5% in this case, because we have a two-point symmetrical distribution).

Similarly, $E(r_B) = 8.5\%, \sigma_B = 6.5\%$ (3 pts)

With Cov(A,B) = 0.008125, plug all these numbers in the above equation, you get $w_A = -1.0833$ and $w_B = 1 - (-1.0833) = 2.0833$ (2 pts)

So, short sell \$108.33 in A, and long \$208.33 in B. (1 pt)

Question 4 cont'd:
(b) There further exists a stock C with an expected return of 3% in recession and 15% in boom. Stock C's current price is $2.

(i) Show that there exists an arbitrage opportunity. Design in detail an arbitrage strategy and show the payoffs in all states of the world (5 marks. You should show clearly that your strategy works to get full credit).

(ii) If A and B are fairly priced, what must happen to C's price? (2 marks)

Answer:
(i) There are many arbitrage opportunities here. I'll show the easiest one.

Note that stock C weakly dominates stock B state by state (B vs. C: 3% vs. 2% in recession, and 15% vs. 15% in boom).

So just buy C and short B. For example, I can buy $1 of C and short $1 of B, the payoff follows:

| | | Terminal Payoff | |
|---|---|---|---|
| | Today | Recession | Boom |
| Buy $1 of C | -$1 | -2% ($1) | -15% ($1) |
| Short $1 of B | +$1 | 3%($1) | +15% ($1) |
| Total | 0 | $0.01 | $0 |

(ii) C's price must increase (so that its return will be smaller to make the arb. profit disappear)

Question 5: 13 marks. This question is split to this and the next page. (**Source: Chapter 8, Question 4**)

Let $R$ denote excess return. The average market excess return (market risk premium) is 0.07 (i.e., 7%). Consider the two CAPM regression results for stocks A and B:

Stock A:

$$\begin{aligned} R_A &= 0.01 + 1.5R_M \\ R_{\text{squared}} &= .426 \\ \sigma_\varepsilon &= 19.3\% \end{aligned}$$

Stock B:

$$\begin{aligned} R_B &= -0.02 + 1.1R_M \\ R_{\text{squared}} &= .376 \\ \sigma_\varepsilon &= 20.1\% \end{aligned}$$

(a) (3 marks) Which stock has more firm-specific risk, and why? And which has greater market risk, and why?

Answer: Firm-specific risk is measured by the residual standard deviation. Thus, stock B has more firm-specific risk: $20.1\% > 19.3\%$. (1.5 pt)

Market risk is measured by beta. A has a larger beta coefficient: $1.5 > 1.1$. (1.5 pt)

(b) (2 marks) For which stock does market movement explain a greater fraction of return variability, and why?

Answer: $R^2$ measures the fraction of total variance of return explained by the market return. A's $R^2$ is larger than B's: $.426 > .376$.

(c) (2 marks) Which stock had an average return in excess of that predicted by the CAPM?

Answer: The average rate of return in excess of that predicted by the CAPM is measured by alpha, the intercept of the SCL. $\alpha_A = 0.01$ which is greater than $\alpha_B = -0.02$.

Question 5 cont'd:

(d) (6 marks) Assume that you collect the data for another 48 stocks, and run a two-pass regression using the 50 stocks to test the validity of the CAPM. Here are the results of your second-pass regression:

| Regressions statistics | | |
|---|---|---|
| R Square | 0.04 | |
| Observations | 50 | |

| | Coefficient | Standard Error |
|---|---|---|
| Intercept | 0.038 | 0.014 |
| Slope | 0.025 | 0.0121 |

Comment on your findings on the validity of the CAPM and on the question of whether investors are rewarded for bearing beta-risk at all.

Answer: (2 pts each) CAPM hypotheses state that (1) intercept = 0, and (2) slope = average market risk premium.

For hypothesis (1), t-stat $= \frac{0.038}{0.014} > 1.96$, or reject the hypothesis at the 5% significance level;

For hypothesis (2), t-stat $= |\frac{0.025-0.07}{0.0121}| > 1.96$, again reject the hypothesis that the slope equals the market risk premium at 5% level;

Finally, whether investors are rewarded for bearing beta risk at all is equivalent to testing whether the slope is equal to zero or not:

t-stat $= \frac{0.025}{0.0121} = 2.06 > 1.96$

i.e. results confirm that investors are rewarded for bearing risk.

(if you state on R-square and make correct comment that the R-squared is too low, you'll get 1 pt; but your total point should be exceed 6)

Question 6: 13 marks. This question is split to this and the next page. Sub-questions (a), (b) and (c) are independent.

(a) (4 marks) ABC Inc. shares are currently selling for a price of \$130. You are optimistic about this firm. Suppose you buy 60 shares at 55% margin. How low can the share price fall before you receive a margin call if the maintenance margin is 35%? If the share price were to drop to this level, how much extra cash would you have to add to your margin account? **(Source: Problem set)** Answer:

$$.35 = \frac{60P - (1 - .55)(60 \times 130)}{60P} \quad \Rightarrow P = \$90$$

Once you receive a margin call, you must inject cash to bring the margin ratio to the initial margin. In this case:

$$.55 = \frac{60 \times 90 + X - (1 - .55)(60 \times 130)}{60 \times 90} \quad \Rightarrow X = \$1{,}080$$

(b) (5 marks) Old Economy Traders opened an account to short-sell 500 shares of Internet Dreams at \$30 per share. The initial margin requirement was 55%. A year later, the price of Internet Dreams has risen from \$30 to \$35, and the stock has paid a dividend of \$1 per share. What's the remaining margin (in dollar value) in the account? If the maintenance margin is 35%, will Old Economy receive a margin call? **(Source: Ch3, Question 4)**

Answer:
The initial margin was $.55 \times 500 \times \$30 = \$8{,}250$.

The firm loses $30 - 35 = \$5$ per share, plus a payment of \$1 dividend pay share, or a total loss of \$6 per share or $\$6 \times 500 = \$3{,}000$ of loss.

Remaining dollar margin: $\$8{,}250 - \$3{,}000 = \$5{,}250$.

Margin ratio: $\frac{5250}{500(35)} < 35\%$. Yes, the firm will receive a margin call.

Question 6 cont'd: (c) (4 marks) Suppose that the characteristics of securities A, B and C are as follows:

| Security | Expected return | Standard deviation |
|---|---|---|
| A | 0.15 | 0.15 |
| B | 0.17 | 0.20 |
| C | 0.05 | 0 |
| Correlation between returns of A and B $= -1$ | | |

If there is no "free lunch", is it possible for Security C to co-exist with Securities A and B with the return patterns shown? **(Source: Ch6, P12)**

Answer:
Since A and B are perfectly negatively correlated, a risk-free portfolio can be created and its rate of return in equilibrium will be the risk-free rate. To find the proportions of this portfolio (with $w_A$ invested in A and $w_B = 1 - w_A$ in B), set the variance equal to zero.

$$\begin{aligned} Var(w_A r_A + w_B r_B) = \sigma_P^2 &= w_A^2 \sigma_A^2 + 2Corr(r_A, r_B)\sigma_A \sigma_B + w_B^2 \sigma_B^2 \\ &= w_A^2 \sigma_A^2 - 2\sigma_A \sigma_B + w_B^2 \sigma_B^2 \\ &= (w_A \sigma_A - w_B \sigma_B)^2 \end{aligned}$$

I.e. $\sigma_P = Abs(w_A \sigma_A - w_B \sigma_B)$

$0 = 0.15 w_A - 0.20(1 - w_A)$

$w_A = 4/7$

The expected rate of return on this risk-free portfolio is:

$$E(r) = .15(4/7) + .17(1 - 4/7) = 0.1586 \neq \text{C's return of } 0.05$$

Therefore, C can not coexist with A & B.

Question 7: 14 marks. This question is split to this and the next page. **(Source: Ch2, Problem 11)**

Consider the three stocks in the following table. $P_t$ represents price at time $t$, and $Q_t$ represents shares outstanding at time $t$. Stock C splits two for one in the last period.

| | $P_0$ | $Q_0$ | $P_1$ | $Q_1$ | $P_2$ | $Q_2$ |
|---|---|---|---|---|---|---|
| A | 90 | 100 | 105 | 100 | 110 | 100 |
| B | 50 | 200 | 55 | 200 | 45 | 200 |
| C | 100 | 200 | 110 | 200 | 60 | 400 |

(a) (2 marks) Calculate the rate of return on a price-weighted index consisting of the three stocks for the first period ($t = 0$ to $t = 1$).

Answer:
Index at $t = 0$

$$\frac{90 + 50 + 100}{3} = 80$$

Index at $t = 1$

$$\frac{105 + 55 + 110}{3} = 90$$

Return: $\frac{90}{80} - 1 = 12.5\%$

(Directly calculate return will receive full mark)

(b) (4 marks) Calculate the price-weighted index for the second period ($t = 1$ to $t = 2$).

Answer:
Divisor:

$$\frac{105 + 55 + 110/2}{\frac{105+55+110}{3}} = 2.3889$$

Index:

$$\frac{110 + 45 + 60}{2.3889} = 89.9996$$

(c) (3 marks) If the starting value-weighted index at time 0 is 1,000, what is the value-weighted index at time 2?

Answer:

$$1000 \times \frac{110(100) + 45(200) + 60(400)}{90(100) + 50(200) + 100(200)} = 1{,}128.21$$

Question 7 cont'd:

(d) (5 marks) You are tracking an (arithmetically) equal-weighted index consisting of stocks A, B and C. You invest $1 each at stocks A, B, and C at the end of time 0 and re-balance your portfolio every two periods. How should you rebanlace your portfolio at the end of time 2 (assume that stocks are perfectly divisible)?

Answer:

| Stock | Time 0 | Time 2 |
|---|---|---|
| A | $1 | $1 (110/90) = 1.22 |
| B | $1 | $1 (45/50) = 0.90 |
| C | $1 | $1 (60*2)/100 = 1.20 |
| Total | | 3.32, so that each stock should have 3.32/3 = $1.11 investment |

Therefore, disregarding rounding error, sell $.11 of A, buy $0.21 of B, and sell $0.09 of C.

Question 8: 11 marks. This question is split to this and the next page. (a) (8 marks) You are designing an optimal portfolio allocation for a client, who has a mean-variance utility:

$$U(r) = E(r) - \frac{1}{2}A\sigma_r^2$$

where $r$ is a random return, $A$ is the risk aversion coefficient, and $\sigma_r^2$ is the return variance. Through a customized questionnaire, you figure out that she has a risk aversion coefficient of 4. You further narrow down her choice of risky assets to Funds A, B, and C, which have the following estimated statistics of annual returns:

| | | Covariance Matrix | | |
|---|---|---|---|---|
| | $E(r_i)$ | A | B | C |
| A | 0.17 | 0.09 | 0.045 | 0 |
| B | 0.11 | 0.045 | 0.06 | 0 |
| C | 0.13 | 0 | 0 | 0.07 |

Through the help of a Morningstar tool, you figure out that your client's tangency portfolio should be: 49.26% in A, 4.77% in B, and 45.97% in C. The riskfree T-bill pays 4% per annum. What's the optimal portfolio allocation for the client?

Answer:

$$E(r_p) = 0.4926(0.17) + 0.0477(0.11) + 0.4597(0.13) = 0.1488$$

$$\sigma_P^2 = 0.4926^2(0.09) + 0.0477^2(0.06) + 0.4597^2(0.07) + 2(0.4926)(0.0477)(0.045) = 0.0389$$

Plug these two numbers in the formula:

$$\begin{aligned} y &= \frac{E(r_p) - r_f}{A\sigma_p^2} \\ &= \frac{0.1488 - 0.04}{4 \times 0.0389} \\ &= 0.6992 \end{aligned}$$

Therefore, the optimal portfolio allocation weights are:

A: 0.6992 (49.26%) = 34.09%
B: 0.6992 (4.77%) = 3.34%
C: 0.6992 (45.97%) = 32.14%
Rf: $1 - 0.6992 = 30.08\%$

Question 8 cont'd:
(b) (3 marks) Give three reasons why the optimal allocation that you designed in (a) may not be the same for another client.

Answer:
All of the elements that lead to the optimal allocation of assets can be difference across investors. Examples:

(1) preferences—people may have different preferences, e.g., constraining the investment universe to less risky assets only;
(2) constraints—e.g. short-sales and borrowing are not allowed;
(3) different beliefs about the expected return and variance of securities;
(4) investors may not be mean-variance optimizers, e.g., skewness
(5) risk aversion can be different;
etc.