AFM 271

Midterm Examination #1

Friday June 05, 2009

Prof. J. Thompson

Name: _____

Student Number: _____ **Section Number:** _____

Duration: 2 hours

Instructions:

1. **Answer all questions in the space provided.**
2. **Show all of your calculations.**
3. **The examination has 11 pages (not including this cover page). Verify that your copy is complete.**
4. **Materials allowed: calculator.**
5. **Unless specifically instructed otherwise, provide final answers relating to percentage rates to four decimal places (e.g. 6.27% or .0627) and provide final answers involving dollar amounts to two decimal places (e.g. $98.27).**
6. **To have your exam considered for re-grading, the exam must be written in ink.**
7. **Page 10 is for rough work only. DO NOT write anything on this page that you want graded. You are not expected to hand this page in.**
8. **Page 11 of the exam is a formula sheet. Do not write any part of your answers on this page. It will not be graded. If you find it easier, you can detach this from your exam. You are not expected to hand in the formula sheet.**

<u>Mark Distribution</u>

1. _____/19

2. _____/17

3. _____/15

4. _____/20

5. _____/17

6. _____/12

Total: _____/100

Question 1: 19 marks.

(a) (5 marks) Suppose that you deposit $12,000 today. How much will your investment be worth after 10 years if your deposit earns:

(i) simple interest at an annual rate of 4%;

(ii) semi-annually compounded interest at a stated annual rate of 4%;

(iii) daily compounded interest at a stated annual rate of 4%?

(i) $\$12{,}000 \times (1 + 10(.04)) = \$16{,}800$ *(1 mark)*

(ii) $\$12{,}000 \times (1 + .04/2)^{2\times 10} = \$17{,}831.37$ *(2 marks)*

(iii) $\$12{,}000 \times (1 + .04/365)^{365\times 10} = \$17{,}901.50$ *(2 marks)*

(b) (6 marks) You want to buy a new sports car from Muscle Motors for $56,000. The contract is in the form of a 48-month annuity due at a rate of 8.15%, compounding monthly. What will your monthly payment be?

Note that since this is an annuity due, the first payment is today. We have:

$$\begin{aligned} \$56{,}000 &= C + C \times \frac{1 - (1 + .0815/12)^{-47}}{.0815/12} \\ \$56{,}000 &= C(1 + 40.1214) \\ C &= \$1{,}361.82 \end{aligned}$$

(c) (8 marks) You have just purchased a $42,000 car. You have financed your purchase with a downpayment of $12,000 and a $30,000 bank loan at an interest rate of 5% compounded annually. The car loan is to be amortized over 5 years. Payments are to be made on a monthly basis (at the end of each month). The first payment will be one month from today.

(i) Calculate the monthly payment on the car loan. (4 marks)

(ii) Calculate the interest portion and the principal portion of the 20th payment. (4 marks)

(i) The monthly interest rate is

$$(1+r)^{12} = 1.05 \Rightarrow r = 1.05^{1/12} - 1 = .00407412378.$$

The monthly payment is then:

$$\begin{aligned} \$30{,}000 &= C \times \left[\frac{1 - 1.00407412378^{-60}}{0.00407412378}\right] \\ \Rightarrow C &= \$564.61. \end{aligned}$$

(ii) The balance owing after the 19th payment is:

$$\$564.61 \times \left[\frac{1 - 1.00407412378^{-41}}{0.00407412378}\right] = \$21{,}279.12.$$

The interest portion of the 20th payment is:

$$\$21{,}279.12 \times .00407412378 = \$86.69.$$

The principal portion of the 20th payment is:

$$\$564.61 - \$86.69 = \$477.92.$$

Question 2: 17 marks.

Randy has income of \$70,000 today (period 0). He will have income of \$100,000 tomorrow (period 1). The market rate of interest is 8%.

(a) (5 marks) What is the present value and future value of Randy's income endowment?

$PV = 70{,}000 + \frac{100{,}000}{1.08} = \$162{,}592.59$ (2.5 marks)
$FV = 70{,}000(1.08) + 100{,}000 = \$175{,}600$ (2.5 marks)

(b) (7 marks) If Randy wants to spend an equal amount today (period 0) as he does tomorrow (period 1), how much can he spend?

We want to find S such that:

$S + \frac{S}{1.08} = 162{,}592.59$

This implies that $S = 84423.08$.

(c) (5 marks) Randy now has an investment opportunity. It requires \$30,000 at period 0 and returns \$35,000. Calculate the NPV of this project. Should he invest in the project? why or why not.

$NPV = \frac{35{,}000}{1.08} - 30{,}000 = 2407.41$

Yes, Randy should invest in the project since it yields positive NPV. This means that putting the \$30,000 into the project is better than the next best alternative for the cash (an investment at 8% interest).

Question 3: 15 marks.

DL Company (DLC) has the following balance sheet for Year 0:

Current assets	$5,000.00	Current liabilities	$6,000.00
Fixed assets:	$18,750.00	Debt	$10,000.00
		Equity:	
		Stock	$5,500.00
		Accumulated retained earnings	$2,250.00
Total assets	$23,750.00	Total liabilities and equity	$23,750.00

Sales are currently $25,000 and are forecast to grow by 8% over the next year. Current assets are expected to be 20% of sales, and current liabilities are expected to be 24% of sales. Fixed assets are forecast to be 75% of sales, and cost of goods sold are forecast to be 80% of sales. DLC can borrow at an interest rate of 8%. Assume that any new borrowing occurs at the start of the year (so that interest payments are determined by this year's debt). DLC faces a corporate tax rate of 40%. The firm has a policy of paying out 35% of after tax profits as dividends and does not intend to issue any new shares of stock. Based on this information, calculate a pro forma income statement and a pro forma balance sheet for next year. Use the next page of the exam to show your supporting calculations.

Income Statement Year 1

Sales	_____
Cost of goods sold	_____
Interest payments	_____
Profit before tax	_____
Taxes	_____
Profit after tax	_____
Dividends	_____
Retained earnings	_____

Balance Sheet Year 1

Current assets	_____	Current liabilities	_____
Fixed assets:	_____	Debt	_____
		Equity:	
		Stock	_____
		Accumulated retained earnings	_____
Total assets	_____	Total liabilities and equity	_____

Calculations for Question 3:

Start by calculating next year's sales and items directly related to this figure:

$$\begin{aligned}
\text{Sales} &= \$25{,}000(1.08) = \$27{,}000.00 \\
\text{Cost of goods sold} &= \$27{,}000(.80) = \$21{,}600.00 \\
\text{Current assets} &= \$27{,}000(.20) = \$5{,}400.00 \\
\text{Current liabilities} &= \$27{,}000(.24) = \$6{,}480.00 \\
\text{Net fixed assets} &= \$27{,}000(.75) = \$20{,}250.00
\end{aligned}$$

Now calculate debt and interest payments:

$$\begin{aligned}
\text{Debt} + \text{Accumulated retained earnings} &= \$25{,}650 - \$6{,}480 - \$5{,}500 = \$13{,}670 \\
\text{Profit before tax} &= \$27{,}000 - \$21{,}600 - (.08)\text{Debt} \\
&= \$5{,}400 - (.08)\text{Debt} \\
\text{Profit after tax} &= \$3{,}240 - (.048)\text{Debt} \\
\text{Retained earnings} &= \$52{,}106 - (.0312)\text{Debt} \\
\text{Accumulated retained earnings} &= \$2{,}250 + \$2{,}106 - (.0312)\text{Debt} \\
\Rightarrow \text{Debt} + \$2{,}250 + \$2{,}106 - (.0312)\text{Debt} &= \$13{,}670 \\
\text{Debt} &= \$9{,}613.96 \\
\text{Interest payments} &= (.08)\$9613.96 = \$769.12
\end{aligned}$$

Pro forma balance sheet:

Current assets	$5,400.00	Current liabilities	$6,480.00
Fixed assets:	$20,250.00	Debt	$9,613.96
		Equity:	
		Stock	$5,500.00
		Accumulated retained earnings	$4,056.04
Total assets	$25,650.00	Total liabilities and equity	$25,650.00

Pro forma income statement:

Sales	$27,000.00
Cost of goods sold	($21,600.00)
Interest payments	($769.12)
Profit before tax	$4,630.88
Taxes	($1852.35)
Profit after tax	$2778.53
Dividends	($972.49)
Retained earnings	$1806.04

Question 4: 20 marks.

Consider the following three government bonds:

Bond	Price	Coupon Rate	Maturity
A	\$1,034.98	6%	3 years
B	\$993.75	4.5%	3 years
C	\$1,009.62	5%	1 years

Assume that each bond has a par value of \$1,000 and that coupon payments are made annually. The first coupon payments will be made one year from today.

(a) (10 marks) Calculate the one year, two year, and three year spot interest rates.

$$r_1 = \left(\frac{1{,}050}{1{,}009.62}\right) - 1 = 4.00\% \quad \text{(2 marks)}$$

$$\text{3 of A: } 3{,}104.94 = \frac{180}{1.04} + \frac{180}{(1-r_2)^2} + \frac{3{,}180}{(1+r_3)^3}$$

$$\text{4 of B: } 3{,}975.00 = \frac{180}{1.04} + \frac{180}{(1-r_2)^2} + \frac{4{,}180}{(1+r_3)^3}$$

$$\Rightarrow 870.06 = \frac{1{,}000}{(1+r_3)^3}$$

$$\Rightarrow r_3 = \left(\frac{1{,}000}{870.06}\right)^{1/3} - 1 = 4.75\% \quad \text{(4 marks)}$$

$$1{,}034.98 = \frac{60}{1.04} + \frac{60}{(1+r_2)^2} + \frac{1{,}060}{1.0475^3}$$

$$\Rightarrow \frac{60}{(1+r_2)^2} = 1{,}034.98 - \frac{60}{1.04} - \frac{1{,}060}{1.0475^3}$$

$$\Rightarrow r_2 = \left(\frac{60}{55.05}\right)^{1/2} - 1 = 4.40\% \quad \text{(4 marks)}$$

(b) (6 marks) Assume that the expectations hypothesis of the term structure holds. What is the one year spot rate expected to be one year from now?

Since the expectations hypothesis holds, the expected one year spot rate must equal our second period forward rate. If this did not hold, then investors will strictly prefer either the shorter or longer maturity bond(s) so that the forward rate would have be different than the expected spot rate to clear the market. (2 marks)

$$E(r_{1,2}) = f_2 = \frac{1.044^2}{1.04} - 1 = 4.80\% \quad \text{(4 marks)}$$

(c) (4 marks) Which of these three bonds will have the lowest yield to maturity? Which will have the highest? Explain why. (Do not calculate the yields to maturity, just provide a brief explanation.)

Bond C will have the lowest yield to maturity. This is because it is a one year zero coupon bond (and so all of its cash flows occur after one year) and the yield curve is rising (i.e. the one year spot rate is the lowest rate). Bond B will have the highest yield to maturity. It pays a lower coupon than A, and so it pays a higher percentage of its cash flows later than A. Since the term structure slopes upwards, it will have a higher yield to maturity than A.

Question 5: 17 marks.

Suppose you take out a $265,000 mortgage at a stated annual interest rate of 5% compounded semi-annually. The payments will be made on a monthly basis (starting in one month) and the amortization period will be 25 years.

(a) (9 marks) What is your monthly payment?

$$\text{rate per month}(1+r)^{12} = (1 = .05/2)^2 \Rightarrow r = 1.025^{2/12} - 1 = .004123915 \quad \text{(4 marks)}$$

$$\Rightarrow 265{,}000 = C \times \left[\frac{1 - 1.004123915^{-300}}{0.004123915}\right]$$

$$\Rightarrow C = \$1{,}541.25 \quad \text{(5 marks)}$$

(b) (8 marks) Calculate the interest portion and the principal portion of the 100th payment.

Balance owing after 99th payment

$$1{,}541.25 \times \left[\frac{1 - 1.004123915^{-201}}{0.004123915}\right] = \$210{,}311.81 \quad \text{(4 marks)}$$

The interest portion of the 110th payment is:

$$210{,}311.81 \times .004123915 = \$867.31 \quad \text{(2 marks)}$$

The principal portion of the 100th payment is:

$$1{,}541.25 - 867.31 = 673.94 \quad \text{(2 marks)}$$

Question 6: 12 marks.

Assess whether each of the following statements is true, false, or uncertain. Justify your answer. All marks are based on the quality of your argument supporting your answer.

(a) (6 marks) If the term structure of interest rates is upward-sloping, then we cannot say whether zero coupon government bonds will have higher or lower yields to maturity than coupon paying government bonds of the same maturity.

False. Yes we can say whether the zero will have a higher or lower YTM than the coupon paying bond. The reason is that the yield to maturity on a bond is a complex weighted average of the spot interest rate at maturity and the spot interest rates for all coupon dates prior to maturity. If the term structure is upward-sloping, then spot rates for coupon dates before maturity are lower than the spot rate for the maturity date, and so the weighted average incorporating these higher rates (i.e. the yield to maturity on the coupon paying bond) will be lower than the spot rate for the maturity date (i.e. the yield to maturity for the zero coupon bond).

(b) (6 marks) Consider the following three government bonds:

Bond	Price	Coupon Rate	Maturity
A	\$1,034.98	5%	3 years
B	\$983.75	3%	3 years
C	\$946.54	0%	1 years

Assume that each bond has a par value of \$1,000 and that coupon payments are made annually. The first coupon payments will be made one year from today. Suppose there is another government bond D which is trading today. Bond D does not pay any coupons, but it will pay its par value of \$1,000 back exactly two years from today. The price of bond D will be higher than the price of bond C.

False. Bond D will not have a higher price than bond C. Bond C pays back \$1,000 in one year, whereas bond D pays back \$1,000 in two years. Since money received farther into the future is worth less in present value terms (since discount rates are non-negative), bond D will be worth less to investors than bond C, and so it will have a lower price.

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Formula Sheet

- Sustainable growth rate:
$$g = \frac{p(1-d)(1+L)}{T - p(1-d)(1+L)}$$
- PV of a single cash flow of C occurring n periods from now:
$$\text{PV} = \frac{C}{(1+r)^n}$$
- FV of an investment of C today at a simple rate of r per period n periods from now (with an interest rate of r per period):
$$\text{FV} = C \times (1 + r \times n)$$
- Effective annual interest rate:
$$\text{EAR} = \left(1 + \frac{\text{SAR}}{m}\right)^m - 1$$
- PV of a single cash flow of C at time T at a continuously compounded annual interest rate r:
$$\text{PV} = C \times e^{-rT}$$
- PV of a perpetuity with first payment of C occurring one period from today (with an interest rate of r per period):
$$\text{PV} = \frac{C}{r}$$
- PV of a perpetuity with first payment of C occurring one period from today and with payments growing at a rate of g per period (with an interest rate of r per period):
$$\text{PV} = \frac{C}{r-g} \qquad (r > g)$$
- PV of an ordinary annuity with first payment of C occurring one period from today and a total of n payments (with an interest rate of r per period):
$$\text{PV} = C \times \left[\frac{1 - (1+r)^{-n}}{r}\right]$$
- PV of a growing annuity with first payment of C occurring one period from today, payments growing at a rate of g per period, and a total of n payments (with an interest rate of r per period):
$$\text{PV} = C \times \left[\frac{1 - \left(\frac{1+g}{1+r}\right)^n}{r-g}\right] \qquad (r \neq g)$$