1. At the beginning of January 2000, Richard Poore deposited \$10,000 in a tax-free savings account that paid 6% interest, compounded monthly.

(a) How much did Richard have at the beginning of July, 2002?

$$A = P(1+i)^n$$

n = # compounding periods

Jan - 2000 → Jan 2002

= 24 months + 6

= 30

$P = 10{,}000$

$i = 0.06$ per annum

$= \left(\frac{0.06}{12}\right)$ per month

4

$$A = 10{,}000\left(1+\frac{0.06}{12}\right)^{30}$$

$$= \$11{,}614.00$$

1 formula

2 correct sub-in

1 ans.

(b) In what year will Richard be able to withdraw \$20,000?

$A = 20{,}000$; $P = 10{,}000$

$1 + i = 1.06$.

Find n in years.

4

3 calculations

1 ans.

$$20{,}000 = 10{,}000(1.06)^n$$

$$\Rightarrow 2 = (1.06)^n$$

$$\Rightarrow \ln(2) = n\ln(1.06)$$

$$\Rightarrow n = \frac{\ln(2)}{\ln(1.06)} \doteq 11.58$$

∴ In 11.58 years, or in mid 2011, he has \$20,000.

OR we may use the 'rule of 70' since this is a doubling time question $\Rightarrow t = \frac{70}{6} \approx 11.67$ years.

2. For each of the following functions, find y'.

(a) $y = 7^{(x-2)(x+3)}$

$$y' = 7^{(x-2)(x+3)} \ln(7) \frac{d}{dx}(x-2)(x+3)$$

3 calculations
4 1 ans.

$$= 7^{(x-2)(x+3)} \ln(7)(x+3+x-2)$$

$$= (2x+1)\, 7^{(x-2)(x+3)} \ln(7)$$

(b) $y = (x \ln x)^2$

$$y' = 2(x \ln x) \frac{d}{dx}(x \ln x)$$

4
3 calculations
1 ans.

$$= 2(x \ln x)(\ln x + 1)$$

OR $2x(\ln x)^2 + 2x \ln x$

(c) $y \ln x = xe^y$

$$\frac{d}{dx} y \ln x = \frac{d}{dx} xe^y$$

4
3 calculations
1 ans.

$$\Rightarrow y' \ln x + \frac{y}{x} = e^y + xe^y y'$$

$$y'(\ln x - xe^y) = \frac{xe^y - y}{x}$$

$$\Rightarrow y' = \frac{xe^y - y}{x(\ln x - xe^y)}$$

There are several correct forms of this answer.

3. Find $\frac{d^2y}{dx^2}$ if $y = e^{2x-y}$.

/5

$$\frac{dy}{dx} = e^{2x-y}\frac{d}{dx}(2x-y)$$

$$\therefore \frac{dy}{dx} = e^{2x-y}\left(2 - \frac{dy}{dx}\right)$$

$$\therefore \frac{dy}{dx} = 2y - y\frac{dy}{dx}$$

3 y'; 2 y''

$$\frac{dy}{dx}(1-y) = 2y \Rightarrow \frac{dy}{dx} = \frac{2y}{(1+y)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{2y'(1+y) - 2yy'}{(1+y)^2}$$

$$= \frac{2y'}{(1+y)^2} = \frac{2}{(1+y)^2}\cdot\frac{2y}{(1+y)} = \frac{4y}{(1+y)^3}$$

4. Find relative minimum and maximum values, and the critical numbers at which they occur, for the following functions:

(a) $y = x^{2/3}(x+1)$ $= x^{5/3} + x^{2/3}$

$$\Rightarrow y' = \frac{5x^{2/3}}{3} + \frac{2x^{-1/3}}{3} = \frac{5\sqrt[3]{x^2}}{3} + \frac{2}{3\sqrt[3]{x}}$$

/5 critical #'s: y' DNE at $x=0$

$y' = 0 \Rightarrow x = -2/5$

1 y'

2 #'s

2 answers

we test critical #'s: $y(0) = 0$; y

$y\left(\frac{-2}{5}\right) = \frac{3}{5}\sqrt[3]{\frac{4}{24}} > 0$

$\therefore$ min at $(0,0)$; max at $(-2/5, y(-2/5))$

since $f''(-2/5) < 0 \Rightarrow$ a max.

(b) $y = \frac{x^2}{2-x}$

$$y' = \frac{2x(2-x) - x^2(-1)}{(2-x)^2} = \frac{4x - 2x^2 + x^2}{(2-x)^2}$$

/5

$$= \frac{4x - x^2}{(2-x)^2} = \frac{x(4-x)}{(2-x)^2}$$

critical #'s:

y' DNE at $x=2$

$y' = 0$ at $x = 0, 4$.

1 y'

1 #'s

1 1st/2nd deriv. test.

2 final ans.

Testing: $y(2)$ DNE. no relative extrema

$y(0) = 0$

$y(4) = \frac{16}{-2} = -8$

and $f''(4) < 0 \Rightarrow$ max.

max at $(4,-8)$

min at $(0,0)$

5. Given the function $f(x) = \dfrac{x^2 - 1}{x^3}$,

(a) Find the x-intercepts and y-intercepts of $f(x)$ if each exists.

$/2$

1 x-int
1 y-int

$y = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

$x = 0 \Rightarrow f(0) = \frac{-1}{0}$, D.N.E. no y-int.

(b) Find the horizontal and vertical asymptotes of $f(x)$ if each exists.

$/2$

1 HA
1 VA

$\lim_{x \to 0^+} f(x) = -\infty$; $\lim_{x \to 0^-} f(x) = \infty \Rightarrow x = 0$ a V.A.

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \dfrac{1/x - 1/x^3}{1} = 0 = \lim_{x \to -\infty} f(x) \Rightarrow y = 0$ a H.A.

(c) Find the relative minimum and maximum values, and the critical numbers at which they occur, for $f(x)$, with intervals of increase and decrease.

2 incr/decr.
2 crit. #'s
2 ans.
$/6$

$f'(x) = x^3(2x) - (x^2-1)(3x^2) = \dfrac{2x^4 - 3x^4 + 3x^2}{x^6}$

$= \dfrac{-x^4 + 3x^2}{x^6} \left(= \dfrac{-x^2 + 3}{x^4} \right)$ critical #'s: $0, \pm\sqrt{3}$

critical points: $(-\sqrt{3}, -\frac{2}{3}\sqrt{3})$, $(\sqrt{3}, \frac{2}{3}\sqrt{3})$, $\cancel{(0, \text{D.N.E.})}$

increase: $(-\sqrt{3}, \sqrt{3})$

decrease: $(-\infty, -\sqrt{3})$, $(\sqrt{3}, \infty)$

$\Rightarrow$ rel. min $(-\sqrt{3}, -\frac{2}{3}\sqrt{3})$; rel. max: $(\sqrt{3}, \frac{2}{3}\sqrt{3})$

(d) Find the points of inflection of $f(x)$, with intervals of concavity.

2 f''
2 intervals $/6$
2 inflec.

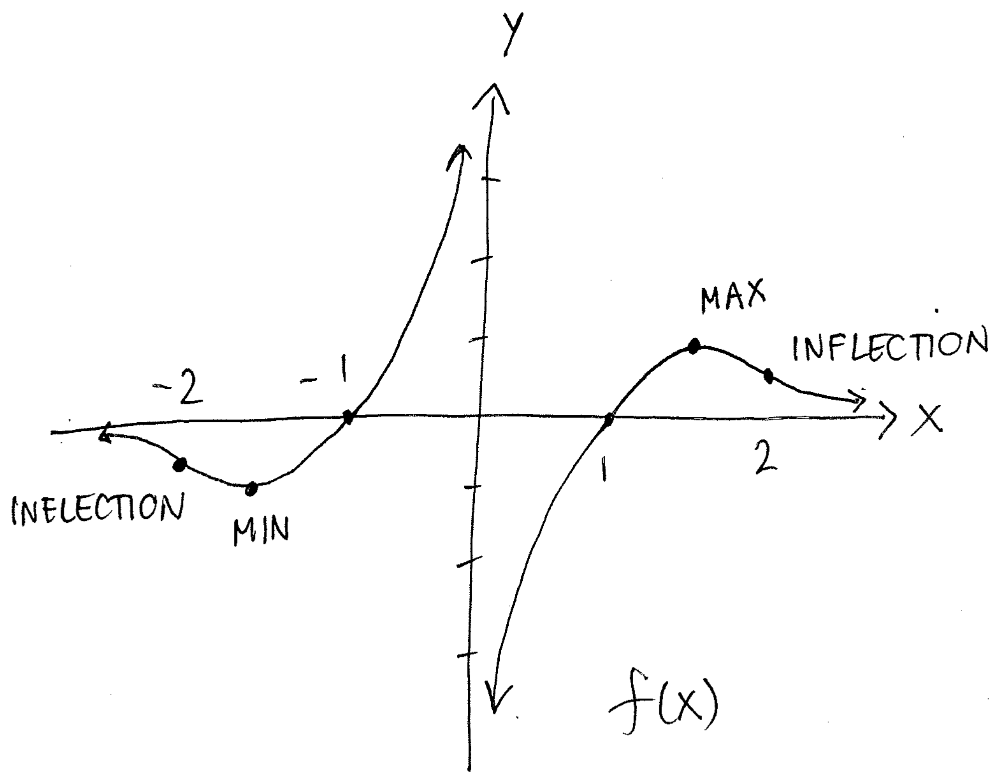
$f''(x) = \dfrac{2x^2 - 12}{x^5}$. $f''(x) = 0 \Rightarrow x = \pm\sqrt{6}$.

$f(-\sqrt{6}) = -\frac{5}{6}\sqrt{6}$, $f(\sqrt{6}) = \frac{5}{6}\sqrt{6}$

CU: $(-\sqrt{6}, 0)$, $(\sqrt{6}, \infty)$.

CD: $(-\infty, -\sqrt{6})$, $(0, \sqrt{6})$

(e) Sketch the graph of $f(x) = \dfrac{x^2 - 1}{x^3}$.



/4

1 shape
1 labels
2 identification of
intercepts, extrema, inflection.

6. Suppose the total cost, in dollars, to produce a product is given by $c(q) = 0.05q^2 + 5q + 500$.

(a) Find the marginal cost, and evaluate it when $q = 10$.

$$c'(q) = 2(0.05)q + 5 = 0.1q + 5$$

4

$$c'(10) = 0.1(10) + 5 = 1 + 5 = \$6.00$$

2 $c'(q)$, 2 $c'(10)$. *need units*

(b) Without using a calculator, estimate the total cost when $q = 11$. Show your work!

$c(10) + c'(10)$ is approximately $c(11)$.
by def'n. of marginal cost.

/5

$$\Rightarrow 0.05(100) + 5(10) + 500 + 6 \approx c(11)$$

$$\Rightarrow 5 + 50 + 500 + 6 \approx c(11)$$

$$c(11) \approx \$561.00$$

2 $c(10)$, 2 for "+6"; 1 final ans.

(c) Find the average cost, and evaluate it when $q = 10$.

$$\bar{c}(q) = \frac{c(q)}{q} = 0.05q + 5 + \frac{500}{q}$$

/4

$$\bar{c}(10) = 0.05(10) + 5 + \frac{500}{10} = 0.5 + 5 + 50$$

$$= \$55.50$$

2 $\bar{c}(q)$; 2 $\bar{c}(10)$

(d) For what level of output will average cost per unit be a minimum?

$$\bar{c}'(q) = 0.05 - \frac{500}{q^2} = 0$$

/7

$$\Rightarrow \frac{500}{q^2} = 0.05 \Rightarrow \frac{q^2}{500} = \frac{1}{0.05} = 20$$

2 $\bar{c}'(q)$

2 #'s

$$\Rightarrow q^2 = 10,000 \Rightarrow q = 100 \quad (q > 0)$$

2 proof a min.

$\bar{c}'(q) < 0$ if $q < 100$
$\bar{c}'(q) > 0$ if $q > 100$
$\Rightarrow$ min. average cost is \$15.00 at 100 units produced.

1 final ans.