Name (Print): ______________________

UW Student ID Number: ______________________

University of Waterloo
Term Test 2
Math 109
Mathematics for Accounting

Date: March 18, 2011

Time: 2:30 p.m. - 4:20 p.m.

Number of pages: 8
(including cover page)

Test type: Closed Book

Additional material allowed:
Non-programmable, non-graphing calcluator.

Circle your section number

Instructor	**Section**	**Lecture Time**
Michelle Ashburner	001	(12:30 p.m. - 1:20 p.m.)
Smith	002	(1:30 p.m. - 2:20 p.m.)

Instructions

1. Write your name and ID number at the top of this page. Please circle your section number up above.
2. Answer the questions in the spaces provided, using the backs of pages for overflow or rough work.
3. Show all your work required to obtain your answers.
4. The most you can earn is 75 points.

FOR INSTRUCTOR'S USE ONLY	
Question	Mark
1	/20
2	/7
3	/15
4	/10
5	/15
6	/8
Total	/75

1. Evaluate the following integrals:

   (a) $\int \frac{\sqrt{t}-7}{t^2}\,dt$

   $\int \frac{\sqrt{t}-7}{t^2}\,dt = \int t^{-3/2} - 7t^{-2}\,dt = -2t^{-1/2} + 7t^{-1} + C = \frac{-2}{\sqrt{t}} + \frac{7}{t} + C$

   Five points : 3 for calculation, 1 for answer, 1 + C.

   (b) $\int_1^2 \frac{e^{\ln x}}{x^2}\,dx$

   $\int_1^2 \frac{e^{\ln x}}{x^2}\,dx = \int_1^2 \frac{x}{x^2}\,dx = \int_1^2 \frac{1}{x}dx = \ln x\,|_1^2 = \ln 2 - \ln 1 = \ln 2.$

   Five points: 3 for calculation, 1 for evaluation at bounds, 1 for answer.

   (c) $\int (x+1)e^x 10^{xe^x}\,dx$

   Let $z = xe^x$, then $dz = (x+1)e^x\,dx$. Hence $\int (x+1)e^x 10^{xe^x}\,dx = \int 10^z\,dz = \frac{10^z}{\ln 10} + C = \frac{10^{xe^x}}{\ln 10} + C$

   Five points : 1 for substitution, 2 for calculation, 1 for answer, 1 + C.

(d) $\int_{-1}^{0} \frac{x^2+4x-1}{x+2}\,dx$

First divide: $\frac{x^2+4x-1}{x+2} = x+2+\frac{-5}{x+2}$. Hence, $\int_{-1}^{0} \frac{x^2+4x-1}{x+2}\,dx$ $= \int_{-1}^{0} (x+2+\frac{-5}{x+2})\,dx = [\frac{x^2}{2}+2x-5\ln|x+2|]\,_{-1}^{0} = -5\ln 2 - (\frac{1}{2} - 2 - 5\ln 1) =$ $-5\ln 2 - \frac{1}{2} + 2 = -5\ln 2 + \frac{3}{2}$

Five points: 1 for division, 2 for calculation, 1 for evaluation at bounds, 1 for answer.

2. Find the area of the region that is bounded by the curves $y = \ln x, x = 0, y = 0$ and $y = 1$.

A sketch shows that this is best solved by horizontal strips, so we change to $x = e^y$ and dy. $\int_0^1 (e^y - 0)\,dy = e^y\,|_0^1 = e^1 - e^0 = e - 1$

Seven points: 2 for changing to dy (or swapping variables to get $y = e^x$), 2 for set up, 2 for calculation, 1 for answer.

3. Marginal cost for a certain process is $c'(q) = \dfrac{100}{\sqrt{3q+72}}$, with fixed costs of \$1000. Marginal revenue for the same process is $r'(q) = 100 - \dfrac{3}{2}\sqrt{2q}$.

(a) Find $c(q)$, the total cost function.

Let $z = 3q + 72$, then $dz = 3dq$. Hence $c(q) = \int \dfrac{100}{\sqrt{3q+72}}\,dq = \dfrac{100}{3}\int \dfrac{1}{\sqrt{z}}\,dz = 2\dfrac{100}{3}\sqrt{z} + C = \dfrac{200}{3}\sqrt{3q+72} + C.$

Since fixed costs are \$1000, $1000 = c(0) = \dfrac{200}{3}\sqrt{72} + C \Rightarrow C = 1000 - \dfrac{200}{3}\sqrt{72} = 1000 - 6\dfrac{200}{3}\sqrt{2} = 1000 - 400\sqrt{2}.$

Thus $c(q) = \dfrac{200}{3}\sqrt{3q+72} + 1000 - 400\sqrt{2}$

Six points: 3 for preliminary integration, 2 for using fixed costs to find the value of C, 1 for answer.

(b) Find $r(q)$, the revenue function.

Let $z = 2q$, then $dz = 2dq$. Hence $r(q) = \int(100 - \dfrac{3}{2}\sqrt{2q})\,dq = \dfrac{1}{2}\int(100 - \dfrac{3}{2}\sqrt{z})\,dz = \dfrac{1}{2}[100z - \dfrac{3}{2}\dfrac{2}{3}z^{3/2}] + C = \dfrac{1}{2}[200q - (2q)^{3/2}] + C = 100q - \sqrt{2}q^{3/2} + C$

Since $0 = r(0) = 100(0) - \sqrt{2}(0)^{3/2} + C$, $C = 0$. Hence $r(q) = 100q - \sqrt{2}q^{3/2}$.

Six points: 3 for preliminary integration, 2 for finding the value of C, 1 for answer.

(c) Find $P(q)$, the function giving the profit for selling q units.

$P(q) = r(q) - c(q) = 100q - \sqrt{2}q^{3/2} - [\dfrac{200}{3}\sqrt{3q+72} + 1000 - 400\sqrt{2}]$
$= 100q - \sqrt{2}q^{3/2} - \dfrac{200}{3}\sqrt{3q+72} - 1000 + 400\sqrt{2}$

Three points: 2 for the formula, 1 for answer.

4. The demand equation for a product is $p = (q-5)^2$ and the supply equation for the same product is $p = q^2 + q + 3$, where p (in hundreds of dollars) is the price per unit when q units are demanded or supplied.

   (a) Find the equilibrium point.

   We set $q^2 + q + 3 = (q-5)^2 = q^2 - 10q + 25 \Rightarrow 10q + q = 25 - 3 \Rightarrow 11q = 22 \Rightarrow q = 2$. When $q = 2$, $p = (2-5)^2 = 9 = 2^2 + 2 + 3$. The equilibrium point is $(2, 9)$.

   Four points: 2 for q, 1 for p, 1 for the equilibrium point.

   (b) Determine consumers' surplus under market equilibrium.

   If the demand function is $f(q)$, then Consumer's Surplus is $CS = \int_0^{q_e} (f(q) - p_e)\, dq = \int_0^2 ((q-5)^2 - 9)\, dq = \int_0^2 (q^2 - 10q + 16)\, dq = [\frac{q^3}{3} - 5q^2 + 16q]_0^2 = \frac{8}{3} - 20 + 32 = \frac{44}{3}$

   Since price is in hundreds of dollars, the final answer is \$1466.67.

   Six points: 1 for formula (even if only implied), 2 for setup, 2 for calculation, 1 for answer.

5. Solve the differential equations.

(a) $\frac{dy}{dx} = \frac{x}{y}$

$\frac{dy}{dx} = \frac{x}{y} \Rightarrow ydy = xdx \Rightarrow \int ydy = \int xdx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C \Rightarrow y^2 = x^2 + D$ or $y = \pm\sqrt{x^2 + D}$

Four points: 1 for separation, 2 for integration, 1 for answer.

(b) $x^2y\prime + \frac{1}{y^2} = 0, y(1) = 2$

$x^2y\prime + \frac{1}{y^2} = 0 \Rightarrow x^2\frac{dy}{dx} = -\frac{1}{y^2} \Rightarrow y^2dy = -\frac{1}{x^2}dx \Rightarrow \int y^2dy = -\int \frac{1}{x^2}dx \Rightarrow \frac{y^3}{3} = \frac{1}{x} + C \Rightarrow y = \sqrt[3]{\frac{3}{x} + D}$

Since $y(1) = 2$, $2 = \sqrt[3]{\frac{3}{1} + D} \Rightarrow 8 = 3 + D \Rightarrow D = 5$. Thus $y = \sqrt[3]{\frac{3}{x} + 5}$

Six points: 1 for separation, 2 for integration, 2 for using initial value to determine the constant, 1 for answer.

(c) A marginal revenue function is given by the differential equation

$$\frac{dr}{dq} = (50 - 4q)e^{-r/5}$$

Find the revenue function.

$\frac{dr}{dq} = (50 - 4q)e^{-r/5} \Rightarrow \int e^{r/5}dr = \int(50 - 4q)dq \Rightarrow 5e^{r/5} = 50q - 2q^2 + C \Rightarrow e^{r/5} = 10q - \frac{2}{5}q^2 + D \Rightarrow \frac{r}{5} = \ln(10q - \frac{2}{5}q^2 + D) \Rightarrow r = 5\ln(10q - \frac{2}{5}q^2 + D)$.

Since $r(0) = 0$, $0 = 5\ln(D) \Rightarrow D = 1$. Hence $r = 5\ln(10q - \frac{2}{5}q^2 + 1)$.

Five points: 1 for separation, 2 for integration, 1 for using initial value to determine the constant, 1 for answer. Two points extra credit if they show that the domain for r is $[0, \frac{25 + \sqrt{635}}{2} \approx 25.0996]$.

6. Use the Trapezoidal Rule and $n = 8$ to approximate $I = \int_0^2 \sqrt[4]{1+x^2}dx$.

$T_n = \frac{\Delta x}{2}[f(x_0)+2f(x_1)+...+2f(x_{n-1}+f(x_n)]$ where $f(x) = \sqrt[4]{1+x^2}$ and $\Delta x = \frac{2-0}{8} = \frac{1}{4}$.
So $T_8 = \frac{1}{8}[f(0)+2f(.25)+2f(.5)+2f(.75)+2f(1)+2f(1.25)+2f(1.5)+2f(1.75)+f(2)] =$
$\frac{1}{8}[1+2(1.015272)+2(1.057371)+2(1.118034)+2(1.189207)+2(1.265220)+2(1.342675)+$
$2(1.419706)+1.495349] = \frac{19.310319}{8} = 2.41379$

Eight points: 1 for Trapezoidal Rule formula (even if implied), 1 for Δx, 1 for correct $f(x)$, 1 for nine correct x_i, 3 for correct evaluations of f at each x_i, 1 for answer.

Extra Credit (5 points): Use Simpson Rule and $n = 8$ to approximate $I = \int_0^2 \sqrt[4]{1+x^2}dx$.

$S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + +2f(x_1) + ... + 4f(x_{n-1} + f(x_n)]$ where $f(x) = \sqrt[4]{1+x^2}$ and $\Delta x = \frac{2-0}{8} = \frac{1}{4}$.
So $S_8 = \frac{1}{12}[f(0)+4f(.25)+2f(.5)+4f(.75)+2f(1)+4f(1.25)+2f(1.5)+4f(1.75)+f(2)] =$
$\frac{1}{12}[1+4(1.015272)+2(1.057371)+4(1.118034)+2(1.189207)+4(1.265220)+2(1.342675)+$
$4(1.419706)+1.495349] = \frac{28.946783}{12} = 2.412232$

Five points: 1 for Simpson's Rule formula (even if implied), 1 for Δx, 1 for correct $f(x)$, 1 for nine correct x_i, 1 for answer.

This page is for rough work. It will not be graded.