Math 136 **Sample Term Test 1**

NOTES: - Questions 4d, 5, 6 on this test cover material that will not be covered on our term test 1.

- Students had 90 minutes to write this test, where you will have 110 minutes.

1. Short Answer Problems

a) List the 3 elementary row operations.

b) Does the spanning set span$\{(1,2,1),(0,1,0),(1,0,1)\}$ represent a line or a plane in $\mathbb{R}^3$? Give a vector equation which describes it.

c) What is the area of the parallelogram induced by $\vec{a}=(2,3)$ and $\vec{b}=(1,-1)$.

d) If A is an $n\times m$ matrix and B is an $m\times p$ matrix, then what is the size of AB?

e) Explain why $\vec{a}\times(\vec{b}\times\vec{c})$ must be a vector in the plane with vector equation $\vec{x}=s\vec{b}+t\vec{c}$, $s,t\in\mathbb{R}$.

2. Consider the system of linear equations:

$$\begin{array}{rcrcrcr} z_1 & - & z_2 & + & iz_3 & = & 2i \\ (1+i)z_1 & - & iz_2 & + & iz_3 & = & -2+i \\ (1-i)z_1 & + & (-1+2i)z_2 & + & (1+2i)z_3 & = & 3+2i \end{array}$$

a) Row reduce the matrix to RREF using elementary row operations.

b) What is the rank of the coefficient matrix?

c) Find the general solution of the system.

3. Let $S=\left\{\begin{bmatrix}1\\2\\1\end{bmatrix},\begin{bmatrix}-1\\3\\2\end{bmatrix},\begin{bmatrix}2\\4\\-1\end{bmatrix}\right\}$.

a) Determine if $\begin{bmatrix}1\\1\\1\end{bmatrix}$ is in the span of S. If so, write it as a linear combination of the vectors in S.

b) Determine if S is linearly independent or dependent.

4. Let $\vec{x} = (1, -2, 1)$, $\vec{y} = (1, 0, -1)$, and $\vec{z} = (2, 3, -1)$. Determine the following.

a) $2\vec{x} - 3\vec{y}$

b) A vector that is orthogonal to $\vec{y}$ and $\vec{z}$.

c) The angle between $\vec{x}$ and $\vec{y}$.

d) $f_A(\vec{z})$ where $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$.

5. Use the definition of linearity to determine which of the following mappings are linear.

a) $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $f(x, y) = (x + y, x^2 + y^2, x^2 - y^2)$.

b) $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (2x - y, x - 2y)$.

6. Determine the standard matrix for the following mappings.

a) $\text{perp}_{(2,1)}$.

b) $\text{DOT}_{(1,-2,3)}$ be defined by $\text{DOT}_{(1,-2,3)}(\vec{x}) = (1, -2, 3) \cdot \vec{x}$

7. Let A be an $m \times n$ matrix. Prove that the system of linear equations $A\vec{x} = \vec{b}$ has a solution for every $\vec{b} \in \mathbb{R}^m$ if and only if the rank of $A = m$.

8. Let $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ be a set of vectors in $\mathbb{R}^3$. Prove that $\operatorname{span} S = \mathbb{R}^3$ if and only if S is linearly independent.