Math 136 **Sample Term Test 1 - 2**

NOTES: - Questions 7, 10b on this test cover material that will not be covered on our term test 1.

1. Short Answer Problems

a) List the 3 elementary row operations.

b) What can you say about the consistency and the number of parameters (free variables) in the general solution of a system of 5 linear equations in 4 variables.

c) What is the area of the parallelogram induced by $\vec{a} = (1, -2)$ and $\vec{b} = (4, -9)$.

d) Let $A = \begin{bmatrix} 3 & 2 & 1 \\ -2 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$. Calculate AB.

e) Let $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ be a set of vectors in $\mathbb{R}^3$. State the definition of the set S being linearly independent.

f) Explain why $\vec{a} \times (\vec{b} \times \vec{c})$ must be a vector in the plane with vector equation $\vec{x} = s\vec{b} + t\vec{c}$, $s, t \in \mathbb{R}$.

2. Consider the system of linear equations:

$$\begin{array}{rcrcrcr} -2x & + & 3y & + & 3z & = & -9 \\ 3x & - & 4y & + & z & = & 5 \\ -5x & + & 7y & + & 2z & = & -14 \end{array}$$

a) Write the augmented matrix and row reduce it to RREF using elementary row operations.

b) What is the rank of the coefficient matrix?

c) Find the general solution of the system.

3. Let $S = \{(1, 2, 1, 1), (2, 5, 4, 2), (-1, -3, -3, 0)\}$. Determine if $(2, 1, 3, 0)$ is in the span of S. If so, write it as a linear combination of the vectors in S.

4. Let $A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 1 \end{bmatrix}$. Write the system of linear equations represented by

$$A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$$

5. Determine if the span of $\{(5,1,0),(3,-1,2),(1,1,-1)\}$ represents a line or a plane in $\mathbb{R}^3$ and give a vector equation which describes it.

6. Use a projection to find the distance from the point $(1,-2,3)$ to the plane $2x_1 - 3x_2 - 5x_3 = 5$.

7. Let $\text{CROSS}_{(1,-2,-1)}$ be defined by $\text{CROSS}_{(1,-2,-1)}(\vec{x}) = (1,-2,-1) \times \vec{x}$.

a) Determine the domain and codomain of $\text{CROSS}_{(1,-2,-1)}$.

b) Prove that $\text{CROSS}_{(1,-2,-1)}$ is a linear mapping.

c) Find the standard matrix of $\text{CROSS}_{(1,-2,-1)}$.

8. Let $S = \{\vec{v}_1, \dots, \vec{v}_k\}$ be a set in $\mathbb{R}^n$.

a) Prove that if S is linearly independent, then $k \leq n$.

b) Prove that if $\operatorname{span} S = \mathbb{R}^n$, then $k \geq n$.

9. Let $B = \{\vec{v}_1, \dots, \vec{v}_k\}$ be a linearly independent set in $\mathbb{R}^n$. Prove that if $\vec{v} \notin \operatorname{span} B$, then $\{\vec{v}_1, \dots, \vec{v}_k, \vec{v}\}$ is linearly independent.

10. Determine whether each of the following statements is true or false. Justify your choices with reasoning based on theorems, definitions, or counter-examples. Answers with no explanations will receive no marks.

a) The only 2×2 matrix A such that $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

b) If $L : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear mapping with $L(0,1) = (1,1)$ and $L(0,1) = (2,2)$, then $L(x,y) = (x+2y, x+2y)$ for all $(x,y) \in \mathbb{R}^2$.

c) A homogeneous system of 4 linear equations in 3 unknowns cannot have infinitely many solutions.

d) Let $\vec{a}$ be any non-zero vector in $\mathbb{R}^n$. The set $\{\vec{a}, \operatorname{proj}_{\vec{a}}(\vec{x})\}$ is linearly dependent for any $\vec{x} \in \mathbb{R}^n$.