NOTE: The questions on this exam does not exactly reflect which questions will be on this terms exam. That is, some questions asked on this exam may not be asked on our exam and there may be some questions on our exam not asked here.

1. Short Answer Problems

a) State the second derivative test.

b) Does the mapping $F(x, y) = (y - x^2, y + x^2)$ have an inverse transformation in a neighbourhood of $(1, 2)$? Justify your answer

c) Find the derivative matrix of the mapping $F(x, y) = (x^2 \sin y, xy^2)$.

2. Let $f(x, y) = \dfrac{x}{y} - \dfrac{y}{x}$.

a) Find the equation of the tangent plane to the surface $z = f(x, y)$ at $(1, 1, 0)$.

b) Use the 2nd degree Taylor Polynomial to approximate $f(1.1, 0.9)$.

3. Consider the function $f(x, y, z) = \ln(x + e^{yz})$.

a) Write the definition of the directional derivative.

b) In what direction(s) does f have a rate of change of -1 at the point $(0, 1, 0)$?

c) Is there a direction in which f has a rate of change of 2 at the point $(0, 1, 0)$? Justify your answer.

4. Consider the function $f(x, y) = \begin{cases} \frac{xy^2}{x^2+2y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases}$

a) What is the domain of f?

b) Where is f continuous on its domain?

c) Where is f differentiable on its domain?

5. Let $u = x^3 f\left(\dfrac{y}{x}, \dfrac{z}{x}\right)$. Show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 3u$.
State any assumptions you need to make about f.

6. Use the method of Lagrange multipliers to prove that if $x^2 + y^2 + z^2 = 1$ then $x^2y^2z^2 \leq \frac{1}{3^3}$.

7. Find the maximum and minimum points of the function $f(x, y) = xye^{x+2y-2}$ on the triangle with vertices $(0, 0)$, $(0, 1)$ and $(2, 0)$.

8. Evaluate $\displaystyle\iint_D \frac{1}{\sqrt{x^2 + y^2}}\, dA$ where D is the region that is inside $x^2 + y^2 = x$ and outside $x^2 + y^2 = \frac{1}{4}$.

9. Evaluate $\displaystyle\iiint_D z\, dV$, where D is bounded by $x = 0$, $x = 1$, $z = 0$, $y + z = 2$, and $y = z$.

10. Evaluate $\displaystyle\iint_{D_{xy}} x^2\, dA$, where D_{xy} is bounded by the ellipse $5x^2 + 4xy + y^2 = 1$.

11. Find the volume of the region bounded by $(x^2 + y^2 + z^2)^2 = x$.

Math 237 **Final F07 Answers**

1. a) See text.

b) Yes by inverse mapping theorem since the Jacobian is non-zero and has continuous partial derivatives.

c) $\begin{pmatrix} 2x\sin y & x^2\cos y \\ y^2 & 2xy \end{pmatrix}$.

2. a) $z = 2x - 2y$. b) 0.4

3. a) See text. b) $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$, c) no.

4. a) $\mathbb{R}^2$. b) $\mathbb{R}^2$. c) $(x, y) \neq (0, 0)$.

5. Verify. We need f differentiable.

6. Prove (max occurs at $(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}})$).

7. Max $\frac{1}{2}$ at $(1, 1/2)$, Min 0 at $(x, 0)$ $0 \leq x \leq 2$ and $(0, y)$ $0 \leq y \leq 1$.

8. $\sqrt{3} - \frac{\pi}{3}$

9. $\frac{1}{3}$

10. $\frac{\pi}{4}$.

11. $\frac{\pi}{3}$.

Math 237 **Final W08 Answers**

1. a) See text.

b) See text.

c) Yes by the extreme value theorem.

2. a) Unit disc $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$.

3. a) $e^2 + 2e^2(x - 1) + 2e^2(y - 1)$

4. a) $z_{xx} = [2\sin v \cdot (2x+y) + 2u\cos v \cdot (3x^2y^3)] \cdot (2x+y) + [2u\cos v \cdot (2x+y) - u^2\sin v(3x^2y^3)] \cdot (3x^2y^3) + 4u\sin v + 6xy^3u^2\cos v$.

5. (0,0) is a local min, $(\pm\sqrt{2}, -1)$ are both saddle points.

6. Max 3, min $-\frac{5}{9}$.

7. a) $-2u$, c) $e - 1$.

8. a) $\frac{35}{2}$, b) $\frac{e^4-1}{4}$.

9. a), b) $\frac{1}{3}$.

10. $\frac{2\pi}{15}$.