NOTE: The questions on this exam does not exactly reflect which questions will be on this terms exam. That is, some questions asked on this exam may not be asked on our exam and there may be some questions on our exam not asked here.

1. Short Answer Problems

a) Let $f : \mathbb{R}^2 \to \mathbb{R}$. What is the greatest rate of change of f from a point (a, b) and in which direction does it lie?

b) Let $G : \mathbb{R}^2 \to \mathbb{R}^2$ and $F : \mathbb{R}^2 \to \mathbb{R}^2$. State the Chain Rule in matrix form.

c) Does the function $f(x, y) = y\cos(e^{xy}) + \frac{\ln(|y|+2^x)}{x^2+1}$ have an minimum on the unit disc $x^2 + y^2 \leq 1$?

2. Let $f(x, y) = \sqrt{1 - x^2 - y^2}$

a) Sketch the domain of f and state the range of f.

b) Draw the level curves $C = f(x, y)$ for $C = 0$ and $C = 1$.

3. Let $f(x, y) = e^{x^2+y^2}$.

a) Find the linear approximation $L_{(1,1)}(x, y)$ of f at $(1, 1)$.

b) Use Taylor's theorem to show that the error in the linear approximation $L_{(1,1)}(x, y)$ is at most $5e^2[(x - 1)^2 + (y - 1)^2]$ if $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

4. Let $z = u^2 \sin v$, $u = x^2 + xy + y^2$ and $v = x^3y^3$. Find $\frac{\partial^2 z}{\partial x^2}$.
Express your answer as a function of u, v, x and y.

5. Let $f(x, y) = x^2 + y^2 + x^2y + 4$. Find and classify the critical points of f.

6. Find the maximum and minimum values of $f(x, y) = x^2y^2 - 2xy$ on the region $x^2 + xy + y^2 \leq 1$.

7. Let D be the region in the xy-plane enclosed by the lines $y = 2 - x$, $y = 4 - x$, $y = x$ and $y = 0$. Let $(x, y) = F(u, v) = (u + uv, u - uv)$.

a) Find the Jacobian of F and show that it is never 0 on D.

b) Sketch the image of D in the uv-plane.

c) Use the mapping F to evaluate $\displaystyle\iint_D \frac{e^{\frac{x-y}{x+y}}}{x + y}\, dx\, dy$.

8. Evaluate the following integrals.

a) $\displaystyle\iint_D xy^2\, dA$, where D is the region $-1 \leq x \leq 2$, $-2 \leq y \leq 3$.

b) $\displaystyle\int_0^1 \int_{2x}^2 e^{y^2}\, dy\, dx$.

9. Let D be the region in $\mathbb{R}^3$ which lies in the first octant and is bounded by the surfaces $y^2 + z^2 = 1$, $y = x$, $z = 0$ and $x = 0$.

a) Find the volume of D using Cartesian coordinates.

b) Find the volume of D using Cylindrical coordinates.

10. Find the volume bounded by the surface $(x^2 + y^2 + z^2)^2 = 2z(x^2 + y^2)$.

Math 237 **Final F07 Answers**

1. a) See text.

b) Yes by inverse mapping theorem since the Jacobian is non-zero and has continuous partial derivatives.

c) $\begin{pmatrix} 2x\sin y & x^2\cos y \\ y^2 & 2xy \end{pmatrix}$.

2. a) $z = 2x - 2y$. b) 0.4

3. a) See text. b) $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$, c) no.

4. a) $\mathbb{R}^2$. b) $\mathbb{R}^2$. c) $(x, y) \neq (0, 0)$.

5. Verify. We need f differentiable.

6. Prove (max occurs at $(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}})$).

7. Max $\frac{1}{2}$ at $(1, 1/2)$, Min 0 at $(x, 0)$ $0 \leq x \leq 2$ and $(0, y)$ $0 \leq y \leq 1$.

8. $\sqrt{3} - \frac{\pi}{3}$

9. $\frac{1}{3}$

10. $\frac{\pi}{4}$.

11. $\frac{\pi}{3}$.

Math 237 **Final W08 Answers**

1. a) See text.

b) See text.

c) Yes by the extreme value theorem.

2. a) Unit disc $x^2 + y^2 \leq 1$, $0 \leq z \leq 1$.

3. a) $e^2 + 2e^2(x-1) + 2e^2(y-1)$

4. a) $z_{xx} = [2\sin v \cdot (2x+y) + 2u\cos v \cdot (3x^2y^3)] \cdot (2x+y) + [2u\cos v \cdot (2x+y) - u^2\sin v(3x^2y^3)] \cdot (3x^2y^3) + 4u\sin v + 6xy^3u^2\cos v$.

5. (0,0) is a local min, $(\pm\sqrt{2}, -1)$ are both saddle points.

6. Max 3, min $-\frac{5}{9}$.

7. a) $-2u$, c) $e - 1$.

8. a) $\frac{35}{2}$, b) $\frac{e^4-1}{4}$.

9. a), b) $\frac{1}{3}$.

10. $\frac{2\pi}{15}$.