AFM 271

Midterm Examination #2

Friday July 03, 2009

Prof. J. Thompson

Name: ______________________________

Student Number: ____________________ **Section Number:** __________

Duration: 2 hours

Instructions:

1. **Answer all questions in the space provided.**
2. **Show all of your calculations.**
3. **The examination has 12 pages (not including this cover page). Verify that your copy is complete.**
4. **Materials allowed: calculator.**
5. **Unless specifically instructed otherwise, provide final answers relating to percentage rates to four decimal places (e.g. 6.27% or .0627) and provide final answers involving dollar amounts to two decimal places (e.g. $98.27).**
6. **To have your exam considered for re-grading, the exam must be written in ink.**
7. **Page 12 of the exam is a formula sheet. Do not write any part of your answers on this page. It will not be graded. If you find it easier to consult this page by detaching it from your exam, please do so. You are not expected to hand in the formula sheet.**

Mark Distribution

1. ________/27

2. ________/12

3. ________/12

4. ________/10

5. ________/14

Total: ________/75

Question 1: 27 marks.

(a) (8 marks) You are given the following data for two stocks, *A* and *B*:

	Period 0		Period 1	
Stock	Price Per Share	Number of Shares Outstanding	Price Per Share	Number of Shares Outstanding
A	\$49.25	1.0 million	\$57.90	1.2 million
B	\$35.50	7.0 million	\$31.23	7.5 million

(i) (2 marks) Calculate the rate of return from period 0 to period 1 for a value-weighted index of these two stocks

(ii) (2 marks) Calculate the rate of return from period 0 to period 1 for an equal-weighted index of these two stocks with geometric averaging

(iii) (2 marks) Calculate the rate of return from period 0 to period 1 for a price-weighted index of these two stocks.

(iv) (2 marks) In the context of this question (i.e., your solutions to the above), discuss how and why the value-weighted index rate of return differs from the price-weighted index rate of return.

(i)

$$\frac{\$57.90 \times 1{,}200{,}000 + \$31.23 \times 7{,}500{,}000}{\$49.25 \times 1{,}000{,}000 + \$35.50 \times 7{,}000{,}000} - 1 = 2.0\%$$

(ii)

$$\left(\frac{\$57.90}{\$49.25} \times \frac{\$31.23}{\$35.50}\right)^{1/2} - 1 = 1.70\%$$

(iii)

$$\left(\frac{\frac{\$57.9+\$31.23}{2}}{\frac{\$49.25+\$35.50}{2}}\right) - 1 = 5.17\%$$

(iii) The value-weighted index will be affected more by stock stock B since it has a higher market cap in both periods. Since B's return is negative, this drags down the value-weighted index below that of the price weighted index which is not affected by this issue.

(b) (4 marks) Exactly two years ago, Anderson purchased 100 shares of ZMH Inc. at a price of \$10 per share. One year ago, ZMH paid a divided of \$0.50 per share. At that time, Anderson immediately used all of the money he received from dividends on each of his shares to purchase more shares of ZMH. The price he paid then was \$10.50 per share. Today the firm has just paid a dividend of \$0.50 per share, and the price of the firm's stock is \$13.00 per share.

(i) (2 marks) Calculate Anderson's arithmetic average return over the past two years on this investment.

(ii) (2 marks) Calculate Anderson's geometric average return over the past two years on this investment.

(i) Anderson's share holdings two years ago were worth \$1,000. One year ago, he received dividends of \$50, and he bought $\$50/10.50 = 4.76190$ new shares. His holdings at that time were worth $\$10.50 \times 104.76190 = \$1{,}099.99995$. He received a \$0.50 dividend today which paid $104.76190(0.5) = \$52.38095$.

His investment is presently worth $\$13.00 \times 104.76190 = \$1{,}361.9047 + 52.38095 = 1414.28565$. His return over the first year was $\$1{,}099.99995/\$1{,}000 - 1 = 9.99999\%$. His return over the second year was $\$1414.28565/\$1{,}099.99995 - 1 = 28.5714286\%$. His arithmetic average return is $(9.99999\% + 28.5714286\%)/2 = 19.29\%$.

(ii) Anderson's geometric average return is $(\$1414.28565/\$1{,}000)^{1/2} - 1 = 18.92\%$.

(c) (4 marks) Cal wants to purchase an annuity today which will pay $C_{nominal} = 200$ per year for 5 years starting at $t = 2$. Let i (the nominal interest rate) be 10% and π (the expected inflation rate) be 4%. Solving this problem with your variables in real terms, what is the value today of this annuity (*note that you will only receive marks if you solve this problem in real terms*)?

First, price an annuity that starts tomorrow:

The first cash flow is valued today $\frac{200}{1.04}$.
The real interest rate is given by $1 + r = \frac{1.1}{1.04}$. *(1 mark)*
The real growth rate is given by $1 + g_{real} = \frac{1}{1.04}$. *(1 mark)*
$\frac{200}{1.04} A_r^5 = 758.158$ *(1 mark)*
We need to discount by the interest rate and take account of inflation. The appropriate discount is:
$(1+r) \times \frac{(1+i)}{(1+r)} = (1+i)$ (recall that $(1+\pi) = \frac{(1+i)}{(1+r)}$ from the fisher relation.)
Therefore the price is: $\frac{758.158}{1.1} = 689.23$ *(1 mark)*

(d) (5 marks) Clara's Consulting Company is considering the purchase of some new equipment. The initial cost of the new equipment is \$100,000. Usage of this new equipment will result in after-tax operating costs of \$30,000 per year for four years, at which point the equipment will need to be replaced. The new equipment will replace some old equipment. The old equipment can be salvaged today for \$74,000, after one year for \$38,000, or after two years for \$15,000 (at which point it must be replaced). Maintenance costs on the old equipment will be \$20,000 after one year and \$35,000 after two years. When should the old equipment be replaced? (Assume that the usage of the new equipment will result in no change in the firm's operating revenues, and that the new equipment will be replaced as needed forever. Also assume that the opportunity cost of capital is 12%.)

$$\frac{\$100{,}000 + \$30{,}000A^{4}_{.12}}{A^{4}_{.12}} = \frac{\$191{,}120.48}{A^{4}_{.12}} = \$62{,}923.44$$ *(2.5 marks)*

PV of costs of keeping old equipment for one more year, and then selling:

$$\$74{,}000 + \frac{\$20{,}000 - \$38{,}000}{1.12} = \$57{,}928.57$$ *(1.5 marks)*

The FV (after one year) of this is $\$57{,}928.57 \times 1.12 = \$64{,}880$. Since this exceeds the equivalent annual cost of the new equipment therefore old equipment should be replaced today. (*1 marks*)

(e) (2 marks) Consider the following data in an excel spreadsheet. The data in cells A1 through A3 represent yearly returns from a stock market. The data in cells B1 through B3 represent the squared deviations from the mean of the data in cells A1 through A3. **In cell A5**, write the excel command that will find the number of yearly returns we have (elements from A1 to A3). **In cell A6** write the excel command that will find the summation of the yearly returns (A1 to A3). **In cell A7**, write the excel command to output the arithmetic average of the stock returns (do not actually calculate the average). **In cell B5** write the excel command that will sum the squared deviations from the mean. **In cell B6**, write the excel command that will calculate the variance of our returns (do not actually calculate the variance).

	A	B
1	12.25	0.0000045369
2	23.85	0.01394761
3	0.01	0.01444804
4		
5	=COUNT(A1:A3)	=SUM(B1:B3)
6	=SUM(A1:A3)	=B5/(COUNT(B1:B3)-1)
7	=A6/A7	

(f) (4 marks) ABC Corp. is just about to pay a dividend. Its current earnings per share are \$6.00. The firm has a policy of paying out $2/3$ of its earnings as dividends. Its return on retained earnings is 24%. The opportunity cost of capital for ABC Corp. is 18%. What is the firm's current stock price? How much of this is due to the net present value of growth opportunities?
The growth rate is $.24 \times 1/3 = .08$. The share price is

$$P = \$4.00 + \frac{\$4.00 \times 1.08}{.18 - .08}$$
$$= \$47.20. \text{ (\textit{2 marks})}$$

The no growth value is $\$6/.18 + \$6 = \$39.33$. The difference of $\$47.20 - \$39.33 = \$7.87$ is due to the net present value of growth opportunities. (*2 marks*)

Question 2: 12 marks.

You are considering two mutually exclusive investment projects, A and B. The opportunity cost of capital for each project is 9%. The projects' cash flows are as follows:

	Period 0	Period 1	Period 2
A	-$25,000	$32,000	$0
B	-$40,000	$0	$55,000

(a) (4 marks) Calculate the discounted pay-back for each project. Based on the discounted pay-back rule, which project(s) should you undertake?

	Period 0	Period 1	Period 2	
A	-$25,000	$29,357.79817	$0	*(1 mark)*
B	-$40,000	$0	$46,292.39963	

Payback for project A is $25{,}000/29{,}357.79817 = 0.8516$ periods.
Payback for project B is $1 + 40{,}000/46{,}292.39963 = 1.86407$. *(1.5 marks)*

Since no threshold was set, we cannot say which project we should undertake. If the threshold was greater than 0.8516 periods, then we would take project A. *(1.5 marks)*

(b) (4 marks) Calculate the IRR for each project.

$$IRR_A = \frac{\$32{,}000}{\$25{,}000} - 1 = 28.00\% \text{ (2 marks)}$$

$$IRR_B = \left(\frac{\$55{,}000}{\$40{,}000}\right) - 1 = 17.26\% \text{ (2 marks)}$$

(c) (3 marks) Based on the IRR criterion, which project should you choose?
Due to different scale and timing of cash flows (and the fact that projects are mutually exclusive), incremental cash flows must be used.

	Period 0	Period 1	Period 2
$B-A$	-$15,000	-$32,000	$55,000

let $x=(1+r)$. then the IRR is found by solving the following:

$$\begin{aligned} 13{,}000x^2+32{,}000x=55{,}000 &= 0 \\ x &= \frac{-32{,}000 \pm \sqrt{32{,}000^2-4(15{,}000)(-55{,}000)}}{2(15{,}000)} \\ &= \{1.1252,-3.2586\} \text{ (2.5 marks)} \end{aligned}$$

The negative root is meaningless here, so the IRR is 12.52%. Since this exceeds the opportunity cost of capital of 9%, B should be chosen over A. (*0.5 marks*)

(d) (1 mark) Based on the IRR criterion, if these projects were independent, what should you do (and why should you do it)?

From part (a) we know that they are both acceptable projects so we should take them both on.

Question 3: 12 marks.

Bandy's Candies Ltd. is considering investing in a new lemon candy product line. A machine to produce the lemon candies can be purchased today for \$3,500. The machine will last forever. The firm believes that the outlook for the lemon candy market is fairly certain over the next year, but that there is substantial uncertainty after that period. In particular, if the machine is purchased today, it will produce after-tax net cash flows of \$196.20 in 1 year. However, there is a 49% chance that strong demand will mean that after-tax net cash flows will be \$250 in year 2 (and in every subsequent year). Unfortunately, there is also a 49% chance that weak demand will result in after-tax net cash flows of \$150 in year 2 (and in every subsequent year). Even worse than the weak demand situation, there is 2% chance that one of the key ingredients in the candies will be discovered as poisonous. In this case, the after-tax net cash flows are \$10 in year 2 (and in every subsequent year). Assume that the risk involved in this investment is entirely diversifiable, so that all cash flows can be discounted at the risk free rate of 5%.

(a) (4 marks) Calculate the NPV of purchasing the machine today.

The NPV is:

$$\begin{aligned} \text{NPV} &= -\$3{,}500 + \frac{\$196.20}{1.05} + \left[\frac{.49(\$250) + .49(\$150) + .02(10)}{.05}\right] \times 1.05^{-1} \\ &= \$424. \end{aligned}$$

(b) (5 marks) Suppose that the firm has an option to sell the machine for \$3,100 after one year. What is the value of this option?

Strong Demand: $\frac{250}{.05} = 5000$. Therefore do not sell.

Weak Demand: $\frac{150}{.05} = 3000$. Therefore sell for $\$3,100$.

Poison Discovered: $\frac{10}{.05} = 200$. Therefore sell for $\$3,100$. (*2 marks*)

We can now calculate the NPV with the option to abandon.

$$\begin{aligned} \text{NPV} &= -\$3{,}500 + \frac{\$196.20}{1.05} + [.49(5000) + .51(3100)] \times 1.05^{-1} \\ &= \$525.90476. \text{ (2 marks)} \end{aligned}$$

Therefore, the value of the option to abandon is: $\$424 - \$525.90476 = \$101.90$. (*1 marks*)

(c) (3 marks) Suppose that there is even more uncertainty regarding the lemon candy market. In particular, suppose that if the market turns out to be strong, after-tax net cash flows will be \$300 per year but if the market is weak, after-tax net cash flows will be only \$100 per year (but everything else is the same). How would this affect your answers to parts (a) and (b)? Do not calculate your answers again, instead describe in words how they would change.

Since you abandoned the project in both the weak and the poison cases, you will do the same under this new scenario. When the demand is high, you will receive more cash flows, while in the two bad states of the world you will still abandon. Therefore the NPV with the abandon option will be higher (so part (b) is higher). The NPV for the base case will be the same since the increased uncertainty doesn't affect the expected value (so part (a) is the same).

Question 4: 10 marks.

Your firm is evaluating an investment project. The project would require purchasing some manufacturing equipment today for $1,500,000. The equipment is in CCA class 8 (20%). When this project is over, there will be many other assets in the CCA class. You have forecast that it will have a salvage value after 10 years of $1,800,000 (nominal). The equipment will produce pre-tax operating revenues of $275,000 (real) after one year. This amount will grow by 3% per year in real terms during years 2 through 10. Pre-tax operating expenses are expected to be $120,000 (real) after one year. This amount will grow by 7% per year in nominal terms until the end of year 10. The project will require an investment in working capital of $150,000 today. One third of this amount (i.e. $50,000 (nominal)) will be recovered after 5 years, and the remaining $100,000 (nominal) will be recovered after 10 years. The nominal discount rate is 11%, the corporate tax rate is 36%, and the expected inflation rate is 2.5% per year. What is the NPV of this investment project?

The real discount rate is $1.11/1.025-1=.08292683$. The real growth rate for expenses is $1.07/1.025-1=.04390244$. Then:

Cost of equipment:		-$1,500,000.00
1) PV after-tax operating revenues:	$\$275{,}000(.64)[1-(1.03/1.08292683)^{10}]/[.08292683-.03]$	$1,310,610.74
2) PV after-tax operating expenses:	$-\$120{,}000(.64)[1-(1.04390244/1.08292683)^{10}]/$ $[.08292683-.04390244]$	-$604,569.25
3) PV salvage value:	$\$1{,}800{,}000/1.11^{10}$	$633,932.06
4) PV capital gain tax	$-\$300{,}000(.50)(.36)/1.11^{10}$	-$19,017.96
5) PV perpetual tax shield:	$[\$1{,}500{,}000(.20)(.36)/(.11+.20)](1.055/1.11)$	$331,124.67
6) Working capital:	$-\$150{,}000+\$50{,}000/1.11^{5}+\$100{,}000/1.11^{10}$	-$85108.99
7) PV lost tax shield (using $min(S,C)$)	$1{,}500{,}000(0.36)(0.2)/[0.2+0.11](1/[1.11^{10}])$	-122,696.53
8) NPV:		-$55,725.24

1) *(1.5 marks)*
2) *(1.5 marks)*
3) *(1 mark)*
4) *(1.5 marks)*
5) *(1.5 marks)*
6) *(1.5 marks)*
7) *(1.5 mark)*

Question 5: 14 marks. Assess whether each of the following statements is true, false, or uncertain. Justify your answer. All marks are based on the quality of your argument supporting your answer.

(a) (5 marks) If firms can deduct depreciation for tax purposes on a straight line basis, then we cannot determine whether the break-even sales point calculated on the basis of accounting income will be lower than the break-even sales point calculated on a present value basis. (You may assume that all relevant variables such as unit sales price, unit variable costs, fixed costs, etc. are constant throughout the life of the project.)

False, we can determine that the break even accounting is less than or equal to the present value break-even.
Let I be the initial cost of the firm's investment and let T denote its economic lifetime. Then the accounting break-even point is given by

$$\frac{\text{fixed costs}(1-T_c)+(I/T)(1-T_c)}{\text{contribution margin}} = \frac{(I/T)+\text{fixed costs}(1-T_c)-(I/T)T_c}{\text{contribution margin}}$$

On the other hand, the present value break-even point (given the assumption about constant variables) is

$$\frac{\text{EAC}+\text{fixed costs}(1-T_c)-(I/T)T_c}{\text{contribution margin}}$$

The only difference between these two expressions is the first term in the numerator, i.e. (I/T) (which is the depreciation expense) for the accounting break-even point vs. the EAC for the present value break-even point. Note that

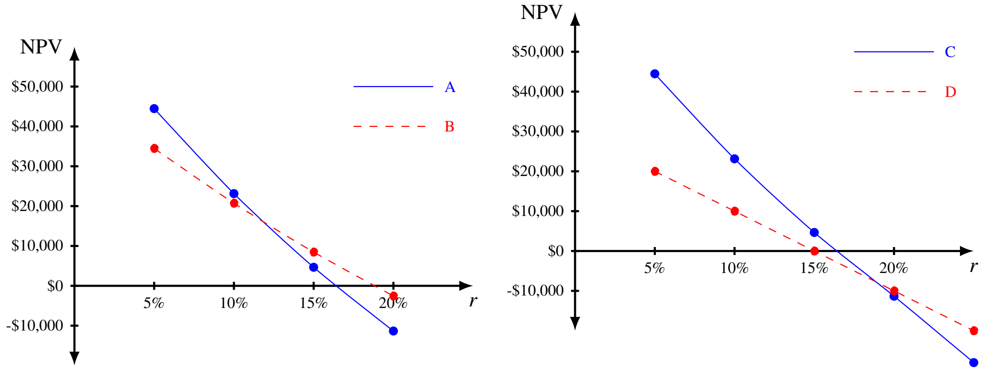
$$\text{EAC} = I/A_r^T.$$

At a discount rate of $r = 0\%$, the present value of an annuity paying \$1 for T periods is simply \$T, so $A_r^T = T$ and the accounting and present value break-even points will be the same. For any $r > 0$, the present value of a T-period annuity will be less than T, i.e. $A_r^T < T$, implying that $\text{EAC} > I/T$ and so the present value break-even point will be higher than the accounting break-even point.

(b) (4 marks) If a firm is acting in the interest of its shareholders, it may not invest in all projects available which will enable it to increase its earnings over time.

True. If the firm is acting in the interest of its shareholders, they should be using NPV or IRR to determine a project. Both of these criterion can rule out projects that may increase the earnings over time. The key variable is the opportunity cost of capital. If this is a standard investing project, then if the opportunity cost of capital is too high, the firm should not pursue, even if it increases their accounting earnings. The important distinction between accounting earnings and NPV and IRR is that accounting earnings doesn't take account of the time value of money.

(c) (5 marks) Consider the following 4 NPV profiles for 4 different projects. Let projects A and B be mutually exclusive and C and D be mutually exclusive. To use the IRR criterion, we must use incremental IRR to determine which projects to pursue or else our NPV rule and our IRR rule may lead to different choices in both cases.



False. Although standard IRR analysis will lead to a potential problem with the comparison between project A and B, it will correspond with NPV when choosing between C and D. The reason for this is that the intersection of C and D occurs where the NPV is negative so that we would not accept the project there anyways.

Midterm #2 Formula Sheet

- PV of a perpetuity with first payment of C one period from today, payments growing at a rate of g per period, PV of a similar annuity with n payments:

$$\text{PV} = \frac{C}{r-g} \qquad (r > g), \qquad \text{PV} = C \times \left[\frac{1 - \left(\frac{1+g}{1+r}\right)^n}{r-g} \right] \qquad (r \neq g)$$

- Roots of a quadratic equation:

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Fisher relation:

$$(1+i) = (1+r) \times (1+\pi)$$

- UCC at end of year n (single asset bought for C at time zero, assuming half-year rule applies):

$$\text{UCC}_n = C(1-d/2)(1-d)^{n-1}$$

- General formula for PV of CCA tax shields (note that ΔUCC will depend on circumstances such as recaptured depreciation, terminal loss, or neither, and that this formula can change depending on whether or not the half-year rule applies):

$$\text{PV} = \frac{CdT_c}{k+d} \times \frac{1+k/2}{1+k} - \frac{\Delta UCCdT_c}{k+d} \times \frac{1}{(1+k)^n}$$

- Accounting break-even point, present value break-even point:

$$\frac{[\text{fixed costs + dep.}](1-T_c)}{\text{contribution margin}}, \qquad \frac{\text{EAC} + \text{fixed costs}(1-T_c) - \text{dep.}T_c}{\text{contribution margin}}$$

- Geometric mean return, unbiased estimate of T year average return forecast:

$$R^g = \left[\prod_{i=1}^{n}(1+R_i)\right]^{1/n} - 1, \qquad R(T) = \frac{T-1}{N-1} \times R^g + \frac{N-t}{N-1} \times R^a$$

- Variance of a single random variable:

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (R_i - \overline{R})^2$$