

Solution Key

Term: **Spring** Year: **2011**

Date of Exam	June 16, 2011
Time Period	Start time: 4:30 pm End time: 6:30pm
Duration of Exam	2 hours
Number of Exam Pages (including this cover sheet):	10
Exam Type (open or closed book)	Closed
Additional Materials Allowed	Formula sheet, both sides of 8.5x11 sheet Calculator

Marking Scheme:

Question	Score
1 12 marks	
2 14 marks	
3 12 marks	
4 12 marks	
5 6 marks	
Total 56 marks	

Question 1: Short Answer Questions (12 marks ~25 min.)

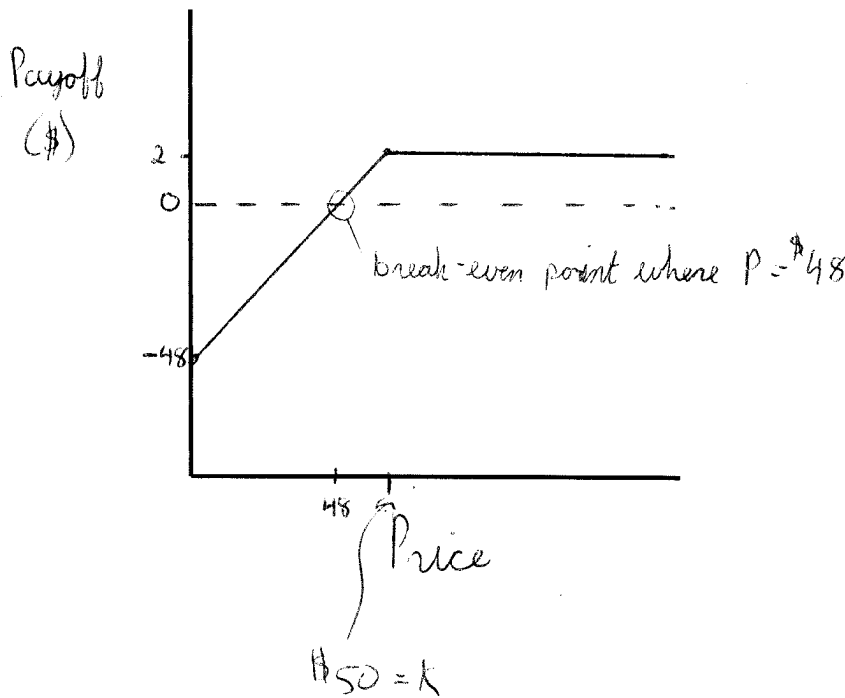
i) (2 marks) Assuming that all 5 bonds have the same yield to maturity, rank the following bonds according to their interest rate sensitivity, from the most to the least interest rate sensitive:

- A. 8 years to maturity and 5% coupon rate (coupons paid annually)
- B. 3 years to maturity and 7% coupon rate (coupons paid semi-annually)
- C. 8 years to maturity and 0% coupon rate (discount aka zero-coupon bond)
- D. 8 years to maturity and 5% coupon rate (coupons paid semi-annually)
- E. 3 years to maturity and 5% coupon rate (coupons paid semi-annually)

2 marks
all or none

Answer: C, A, D, E, B

ii) (4 marks) Draw the payoff function on the axis below from the perspective of the seller of a put option given that the strike price is \$50 and the put premium is \$2. Be sure to label points on the axis corresponding to the inflection point and the break-even point and include axis labels. What is the maximum possible loss of an investor in this position?



axis labels = 1 mark
 $K \neq P$ = 1 mark
 break-even = 1 mark
 max loss = 1 mark

max loss is where $P = 0 \Rightarrow \text{Payoff} = P - k + p = 0 - 50 + 2 = -48$

iii) (6 marks) List and explain the three components of risk as discussed in class.

- 1) Inflation risk : compensation for change in purchasing power between start of investment and payment of cash flows
- 2) Deferred consumption : compensation for having ^{to} delay consumption to a future date
- 3) Default risk : compensation for incurring the risk that expected cash flows will not be realized

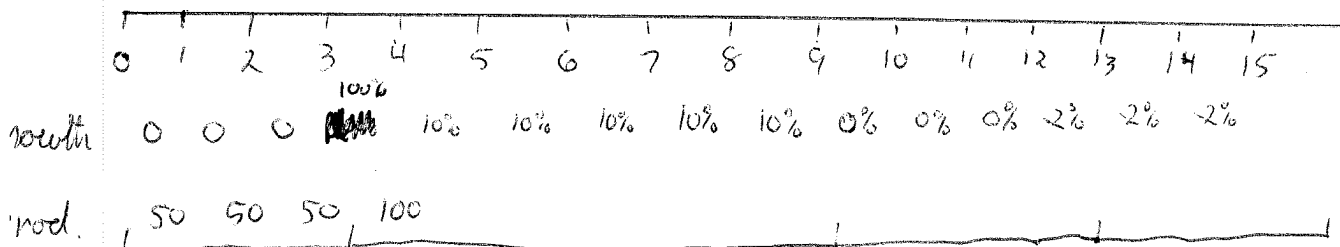
2 marks each, half mark if risk is listed with no explanation.

Question 2: Time Value of Money (14 marks ~30 min.)

White truffles are a prized underground growing mushroom found in the Piedmont region of Northern Italy. The Italians use dogs to locate the wild growing truffles during harvesting. A truffle farmer has compiled the following information regarding truffle dogs.

- Truffle dogs can be expected to live for 15 years
- Truffle dogs require \$3000 per year in upkeep (i.e. food, veterinary and other costs), assume these costs are realized at the start of each year
- It takes three years (from birth to the end of the third year of the dog's life) to train the dog. During this period the dog will locate 50 pounds of truffles per year
- In the fourth year the truffle dog will locate 100 pounds of truffles after which the volume of truffles located by the dog will increase by 10% annually for the next 5 years (i.e. the volume of truffles located in year 5 would be 100×1.1 and so forth)
- Following this period of 10% annual growth, production remains constant for 3 years and then deteriorates at a rate of 2% per year for the remainder of the dog's life.
- Truffles are expected to sell for \$1000 per pound over the next 15 years. Assume the farmer sells all the truffles at the end of each year.
- Assume a discount rate of 8% APR with semi-annual compounding

What is the net present value of a truffle dog born today (i.e. at the start of year 1) based on the information given above?



$$EAR = \left[1 + \frac{APR}{m} \right]^m - 1 = \left[1 + \frac{0.08}{2} \right]^2 - 1 = 8.16\% \quad [2 \text{ marks}]$$

$$NPV = \text{Annuity}_{t=3}^{C=50} + \text{Growing Annuity}_{t=6}^{C_4=100, g=10\%} + \text{Annuity}_{t=3}^{C=10} + \text{Growing Annuity}_{t=3}^{C_{13}=2, g=-2}$$

Continued on the next page.

- Annuity of upkeep

$C = 3000$

$t = 15$

Due

Additional space for Question 2.

$$= \left[50 / 0.0816 \left[1 - \frac{1}{(1.0816)^3} \right] + \frac{100}{0.0816 - 0.1} \left[1 - \frac{1.10}{1.0816} \right]^6 \right] \left[\frac{1}{1.0816^3} \right] + \frac{100(1.10)^5}{0.0816} \left[1 - \frac{1}{1.0816^3} \right] \left[\frac{1}{1.0816^9} \right] +$$

$$\frac{100(1.10)^5 (0.98)}{0.0816 + 0.02} \left[1 - \frac{(0.98)^3}{1.0816} \right] \left[\frac{1}{1.0816^{12}} \right] \times 1000 - \frac{3000}{0.0816} \left[1 - \frac{1}{1.0816^{15}} \right] \left[1.0816 \right]$$

$$= \left[128.4837 + 457.4880 + 204.2873 + \overset{155,2426}{\cancel{307,19339}} \right] 1000$$

- 27504.5047

$$= 917,997.10$$

$$= \cancel{1,160,688.40}$$

2 marks for each annuity

Question 3: Bonds (12 marks ~25 min)

Down Under Labs just issued a bond with the following characteristics:

- 20 year maturity
- Annual frequency coupon payments with a coupon rate of 8% APR with annual compounding
- The bond was issued at par value with a face value of \$1000

Part A (7 marks):

If five years from now the yield to maturity decreased by 2% APR with annual compounding, what would be the 5 year total return to an investor who purchased the bond on the day it was issued and sold it five years later?

Note: you do not need to annualize the return, I am asking for the 5 year return between day 0 and the end of year 5. Also assume that all coupon payments in year 5 have been paid.

$$\text{Coupon} = 0.08 \times 1000 = \$80 \quad \text{EAR} = 8\% \text{ w/ annual coupons}$$

[1 mark]

Price at end of year 5

$$\frac{80}{.06} \left[1 - \frac{1}{(1.06)^5} \right] + \frac{1000}{1.06^5} = \$1194.2450$$

[3 marks]

$$\text{Return} = \text{coupon yield} + \text{cap. gain}$$

$$= \frac{80 \times 5}{1000} + \frac{1194.2450}{1000} - 1$$

$$= 0.40 + 0.1942$$

$$= 59.42\%$$

[3 marks]

Continued on the next page.

Question 3 continued:

Part B (5 marks):

At the very end of year 17 the yield to maturity is 8% EAR. Assume that at this point all coupon payments associated with year 17 have just been paid. What is the duration of the bond? Also provide two interpretations of the duration value you calculate.

3 coupons + face value remain $YTM = \text{coupon rate}$
 $\therefore P = 1000 = FV$

$$\text{Duration} = \frac{\frac{80}{1.08} + 2 \times \frac{80}{1.08^2} + 3 \times \frac{1080}{1.08^3}}{1000} = 2.7833$$

[3 marks]

It will take 2.78 years to recover the initial investment of the value at the end of year 17

A 1% change in interest rates (YTM) will result in a 2.78% inverse change in the bond price.

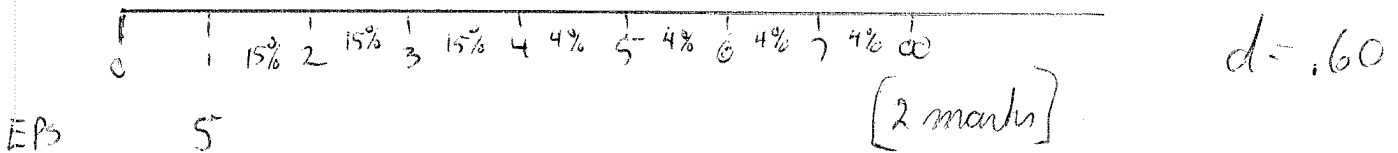
[1 mark each]

Question 4: Equity Valuation (12 marks ~25 min)

Layton Products Inc. is a company that currently pays no dividends.

- Analysts expect the company to report earnings of $EPS_1 = \$5$ per share one year from today and its earnings are expected to grow at an annual rate of 15% over the following three years (until $t = 4$).
- Following this three-year high growth period, earnings will grow at 4% per year forever.
- Layton Products will pay its first dividend four years from today when it will distribute 60% of year-four earnings (i.e. Earnings₄) in the form of dividends. (After the first dividend, the company is expected to maintain the same 60% payout ratio forever.)
- The required rate of return is 11%.

(i) What is Layton Products' share price today?



$$P_0 = \frac{D_4}{r-g} \left[\frac{1}{(1+r)^3} \right] = \frac{EPS_1 (1+g)^3 d}{r-g} \left[\frac{1}{(1+r)^3} \right] = \frac{5(1.15)^3 (.60)}{.11 - .04} \left[\frac{1}{1.11^3} \right]$$

$$= 47.66 \quad [3 \text{ marks}]$$

(ii) What will Layton Products' share price be 10 years from today?

$$P_{10} = \frac{D_{11}}{r-g} = \frac{EPS_{11} d}{r-g} = \frac{EPS_1 (1+g_1)^3 (1+g_2)^7 d}{r-g}$$

$$= \frac{5(1.15)^3 (1.04)^7 (.6)}{.11 - .04}$$

$$= \$85.77$$

Continued on the next page.

[2 marks]

Question 4 continued:

- (iii) What will Layton Products' return-on-equity (ROE) be during its second, slower stage of growth (i.e. the 4% growth stage starting at $t = 4$)?

$$g = ROE \times b \Rightarrow ROE = g/b = .04/(1-.6) = 10\%$$

[1 mark]

Consider this alternative scenario:

- (iv) What will Layton Products' share price be in *ten* years if it starts paying out all of its future earnings in dividends four years from today? (Under the new policy the payout ratio would remain zero from $t = 0$ to $t = 3$ but would rise to 100% at $t = 4$ and stay 100% forever after that.)

$$P_{10} = \frac{EPS_{11}}{r} = \frac{5(1.15)^3(1.04)^7}{.11} = \$90.97$$

[2 marks]

- (v) Would Layton Products' shareholders be better off under this new scenario relative to your result in part on the previous page? Explain **both** from the perspective of relative price and the return on growth opportunities.

Shareholders would be better off. The price increases under the new scenario as more earnings are directed away from a growth opportunity for which $ROE < RRR$. i.e. shareholders are not being adequately compensated for the risk of the growth opportunity.

[2 marks]

Question 5: Financial Statement forecasting (6 marks ~12 min)

Given the following financial statement, calculate the change in net working capital in 2005, 2006 and 2007.

Balance Sheet

	2004	2005	2006	2007
Assets				
Cash and cash equivalents	0	0	0	0
Accounts Receivable	6,025	12,050	12,653	13,285
Property and Equipment	30,655	24,723	19,939	16,081
Other Assets	12,996	10,481	8,453	6,817
Future Income Tax	3,651	5,976	7,905	9,503
Total Assets	53,327	53,230	48,950	45,686
Liabilities and Shareholder Equity				
Accounts Payable	6,922	13,844	14,536	15,263
Long term Debt	11,433	10,692	9,951	9,210
Operating Line	0	3,022	6,508	9,648
Total Debt	18,355	27,558	30,995	34,121
Share Capital	35,534	35,534	35,534	35,534
Retained Earnings	(562)	(9,862)	(17,579)	(23,969)
Total Shareholder Equity	34,972	25,672	17,955	11,565
Total Liabilities and Shareholder Equity	53,327	53,230	48,950	45,686

$$\begin{aligned}\Delta NWC \quad 2005 &= (12050 - 6025) - (13844 - 6922) = -897 \\ 2006 &= (12653 - 12050) - (14536 - 13844) = -89 \\ 2007 &= (13285 - 12653) - (15263 - 14536) = -95\end{aligned}$$

[2 marks for each year]